КОНСОРЦИУМ ЭКОНОМИЧЕСКИХ ИССЛЕДОВАНИЙ И ОБРАЗОВАНИЯ – РОССИЯ И СНГ
ECONOMICS EDUCATION AND RESEARCH CONSORTIUM – RUSSIA AND CIS

V.N. Melnik

STRATEGIC CHOICE OF TRADE POLICY INSTRUMENTS

Final Report

V.N. Melnik

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Resume

What is the optimum mix of trade policy instruments? Usually governments choose among such instruments as quotas, tariffs, explicit of implicit subsidies. The goal of the project is to consider the possibility of a simultaneous use by the government of quotas (and corresponding License fees) and tariffs. The use of quotas and tariffs as complements rather than substitutes allows to carry out a trade policy which dominates – from the efficiency point of view - over a policy based on simple quotas or simple tariffs. The qualitative outcomes of the analysis depend on the type of government (whether it maximizes its revenue or public welfare), market structure and the cost structure of firms operating in the market.

The study aims at improving our understanding of the link between government intervention - the optimal mix of trade policy instruments – and competition in the home goods market.

1. Introduction

Whatever convincing the a reasoning for the benefit of free trade would be, trade barriers still exist. Usually trade barriers appear in the form of the protectionist tariffs, quotas, nontariff barriers and voluntary export restraint (VER). The tariffs for production increase the price, increase internal production reduce consumption, reduce import and create tariff inflows to the state; the quotas on import make the same, but create inflows for the foreign producers, bat not of for the state, which introduced the quotas. Besides the tariffs result in a loss of economy effectiveness, which come from distortion of motivation of behaviour of the domestic producers and customers. On the other hand, the increase of advantage from improving the conditions of trade, which is a consequence of reduced influence of the tariff on the prices of the foreign exporters. In case of "minor" country, which can not influence the prices world markets, the negative effect of the tariff on economy is obvious. The technique of the of the tariff analysis can be applied for study of other aspects of a trade policy: export subsidy, import quotas and VER. The export subsidies result in losses in economy effectiveness. The majority of arguments for the benefit of trade protectionism usually defend special interests.

The given report is devoted to the problems of realization of the state two-part trade policy in case of two countries, two markets, many producers in conditions of imperfect competition, where the usual arguments of free trade are inapplicable. The optimum policy of government is considered from the point of view of two purposes: a maximization of personal incomes; a maximization of domestic welfare. The base model is a two-level game. At the first level governments announce a trade policy, and on the second,- home and foreign producers behave as Cournot competitors.

The report considers:

1) The quotas and tariffs, complementing each other at the optimal two-part trade policy: the import license for an entrance on the market (quota) and payment for unit of import (tariff);
2) The factors which influence the optimum trade policy;
3) The peculiarities of the two-part trade policy in conditions of transition to market;
4) The peculiarity of the two-part trade policy in the internal and external market;
5) The advantages of the modified trade policy.

We have shown that the two-part trade policy **dominates** the simple quota and simple tariff, and the last two mentioned are the special cases of it. Thus the effectiveness of the two-part trade policy depends on the number of competing firms on the market, magnitude of a heterogeneity of functions of costs, degrees of convexity of functions of costs and thus to what firms the given policy will be applied and the governments of which countries will execute the policy. Besides, the effectiveness and outcome of the trade policy we have considered under the following conditions: payoff functions of government G is income of government or W - domestic welfare. We have found conditions at which the optimum two-part trade policy of government will encourage foreign firms to behave as monolithic Stackelberg leader on the second level of the game.

2. The review of the literature.

The classical problem in the theory of the policies of international trade concerns the effect of the tariffs and quotas or the effect of a variety of possible policies. In absolutely competitive models of trade, tariffs and quotas - are usually equivalent, that is the effect of the tariff can be duplicated by the accordingly chosen model of quota. As it is stated by Bhagwati (1965), it will not be true at imperfect competition. He has demonstrated, that the tariffs dominate above the quotas, when there is an imperfect competition in home market. It can be explained by the fact that the keen response of foreign firms at the quota is more exact, than at the tariff. Thus, the quota in comparison with the tariff raises monopolistic force of internal firms. Anderson (1988) and Krishna (1989) generalized the previous results. Anderson considered the duopoly model and showed, that under the certain conditions quotas tend to lower competition. Krishna has shown in model of a Bertrand, that the quota constrains the ability of firms to compete effectively, when the goods are perfect substitutes. Accordingly, we could expect some interesting comparisons between the tariffs and quotas as strategic instrumental means of trade policy. The analysis of the quotas at oligopoly give additional penetrations. Many works uncover the aspects of this problem, for example Itoh and Ono (1984), Harris (1985), Hwang and Mai (1988), Cooper and Riezman (1989), Ishikawa (1994), Krishna (1989), Ries (1993a, b), and others.

There are also works, in which the dominance of the quotas above the tariffs is proved. Some economists considered public models of choice, which analyze the tariffs and quotas. For example, Cassing and Hillman (1985) have shown, that the quotas can dominate above the tariffs. Kaempfer (1989) has proved, that the quotas are more preferable than tariffs, and that exactly quotas can to internal overproduction. Rotemberg and Saloner (1989) used repeated model of the game to show prevalence of the quotas above the tariffs, making of the agreement of the economic agents more difficult.

Perhaps the main difference between tariffs and quotas as policy instruments is related to their effects on foreign firms. Any tariff on foreign firms reduces their profits, and any subsidy to domestic firms also reduces the profits of foreign firms. Quotas, on the other hand, give a much greater possibility for foreign firms to benefit, particularly if the quota is implemented as a voluntary export restraint (VER), meaning that foreign firms keep any quota rents rather than having to buy quota licenses. In effect, VER acts as a device that facilitates a more collusive outcome for foreign firms. This implies that a VER is less likely to be in the interest of a government, which maximizes domestic welfare. A closely related possibility is that VER might lead to a change in the mode of rivalry between firms, as in Harris (1985), who assumes that the imposition of VER at the external free-trade level converts the Bertrand rivalry to a structure in which the domestic firm becomes the Stackelberg price leader.

Eaton and Grossman (1986), using the two-step game model for two firms producing substitute goods, have found, that at the certain conditions of regularity ensuring uniqueness and stability of an equilibrium, the optimum choice will be:

- introducing of the export taxes, if the difference between expectations of the local producer concerning the partner's behaviour and his true behaviour is negative;
- establishing the export subsidies in case the above mentioned difference is positive.

In addition to the choice between the tariffs and quotas, there are many other closely related problems. Even if the attention is limited only to tariffs, there exists a problem either with the announcement of the price or certain tariffs. There should be a choice. As it is shown in Brander and Spencer (1984b), at imperfect competition certain and the ad valorem tariffs – are not equivalent and their relative attractiveness depends on the functional form of demand and other special aspects of the model. In Corden (1971) the wide research of effect of the tariff effect, quotas and other aspects of a trade policy is given. In the work of Rousslang and Suomela (1985) the problem of how the tools of the trade policy can be used in practice is considered.

The author of the project dealt with some interpretations in the modern literature of the modified trade policy (truth not for two-part policy and for simple tariff). So Dixit (1984) has considered the import tariffs, the export subsidy or subsidy for domestic sales. Allowing a subsidy on local sales to shift the emphasis of the analysis away from trade policy, because a government has an incentive to use such a subsidy simply to offset the output-restricting effect of oligopoly. Even in the absence of trade, this obvious intention to subsidize monopolies and oligopoly always exists. Thus application of a two-part trade policy for a case of the simple tariff is justified within the framework of considered models can have additional strategic effects. As far as is justified introductions of the quota on interior firms we can see in the modern literature. So Dixit (1988a) uses a calibration technique to assess the effects of strategic trade policies on the U.S. automobile industry. His underlying model is a reciprocal-markets model with Japanese and American producers where firm conduct is characterized by a conduct parametr model. He focuses on just the U.S. market. Concern about the rising level of Japanese import peneration in the U.S. market led U.S. polisy-market in 1981 to impose a voluntary export restraint (VER) on Japanese imports. On it paths Dixit has received interesting outcomes of comparison of operating policies of USA with optimal policies.

In the given review it is shown, that the quotas and tariffs are different choices. It is supposed in the project to consider a possibility of using the quotas and tariffs as addition to each other, but not as replacement. The two-part trade policy provides for considering the import license for an entrance alongside with the tariff for the unit of import. Such policy was first introduced by Oi (1971) in the classical analysis of a price dilemma of Disneyland. Philips (1983) and Wilson (1993) also considered the examples of nonlinear assigning of the prices. For the case of sole market with homogeneous functions of costs, Fuerst and Kim (1997) have considered the two-part trade policy at complete, but imperfect information. In the market model of one country with homogeneous functions of costs they have found conditions, when the optimal two-part trade policy dominates over the simple tariff and simple quota. It is also fair for the government, which is interested only in maximization of the income, and for benevolent government interested in internal welfare. The effect arises because the large payment for the import license imposes a smaller amount of distortions, than tariff. A government can reach the neutral income, reducing current tariffs and reimbursing the lost incomes by the payment for the license. With inhomogeneous functions of costs Fuerst and Kim (1997) have considered numerical examples, which also have shown advantage of the two-part trade policy.

3. The game structure of strategic trade policy.

The study of strategic trade policy is fundamentally an application of non-cooperative game theory and therefore uses the Nash equilibrium [as first defined by Nash (1950)] as the central equilibrium concept.

The general **noncooperative game with N person** is called the system

$$\boldsymbol{\Gamma = \langle I, \{X_i\}_{i \in I}, \{f_i(x)\}_{i \in I} \rangle}$$

Here I= {1,2,…, N} is a set of players' numbers; $\{X_i\}_{i \in I}$ – a set of strategies the player with number $i \in I$; the collection of strategies $x=(x_1, x_2, \ldots x_N)$, $x_i \in X_i$, is called the situation of the game Γ and $X = X_1 \otimes X_2 \otimes \ldots X_N$ – the set of all situations; for each player $i \in I$, a function f_i: $X \rightarrow R$, called the payoff function of player i.

Each player i selects strategy x_i from strategy set X so as to independently and noncooperatively maximize payoff function $f_i(x_1, x_2, \ldots x_N)$. Let $x^e = (x_1^e, x_2^e, \ldots x_N^e)$ be a feasible vector of strategies, one selected by each player.

The answer to a problem on existence of a Nash equilibrium gives the following theorem:

Theorem(Nash): *Let's assume, what in the game Γ for any player the set of strategies X_i is not empty, compact and convex, and payoff function f_i (.) is concave on x_i and continuous. Then there is a* ***Nash equilibrium****.*

To solve dynamic games with complete information, we use **backward induction**.

4. Two-part trade policy.

Let's consider a base model of the reciprocal markets at two-part trade policy. There are two countries, one is a home country, the other is a foreign one. There are N - of home firms and N* - of foreign firms producing the homogeneous goods. Suppose q_i is a production level of i -home firm for the home market; v_j - level of export of j - a foreign firm for the home market. Accordingly we use a label by an asterisk to designate variables, which are related to the foreign market. Suppose q_i^* - level of export of i home firm for the foreign market; v_j^* - level of production of j foreign firm for the foreign market. Then the complete sales in these two countries equal Q and Q*:

$$Q = \sum_{i=1}^{N} q_i + \sum_{j=1}^{N^*} v_j \quad (4.1) \qquad Q^* = \sum_{i=1}^{N} q_i^* + \sum_{j=1}^{N^*} v_j^* \quad (4.2)$$

The inverse demand curves in a home and foreign country $p(Q), p^*(Q^*)$, where $p, p^* \in C^2$; $p' < 0$, $(p^*)' < 0$. It is also known $c_i\,(q_i)$ - function of costs of i home firm; $c_j^*(q_j^*)$ - function of costs of j foreign firm, where $c_i \in C^2, c_i' > 0, c_i'' > 0$ and $c_j^* \in C^2, (c_j^*)' > 0, (c_j^*)'' > 0$.

The economic result of the firms' activity is determined by the consequent functions of profit:

$$\pi_i = p(Q)q_i - c_i(q_i + q_i^*) + p^*(Q^*)q_i^* - t^*q_i^* - e^*,\ i = 1,...N \tag{4.3}$$

$$\pi_j^* = p^*(Q^*)v_j^* - c_j^*(v_j + v_j^*) + p(Q)v_j - tv_j - e,\ j = 1,...N^* \tag{4.4}$$

Where e, e* - payment for the license to home and foreign government accordingly; t, t*- the tariff per unit of production imposed on home and foreign firms accordingly.

Let's designate by a vector $z = (e, \bar{v}, t)$, $z^* = (e^*, \bar{q}, t^*)$ two-part of trade policies of home and foreign government (here $\bar{q}$, $\bar{v}$ are the quotas on home and import firms). Then it's obvious that $z = (e, \bar{v}, 0)$ - is the simple quota; $z = (0, \infty, t)$ - the simple tariff.

4.1 Trade policy the third-market model.

In a strategic trade policy the third-market model is well known. In this model one or more firms from a domestic country and one or more firms from a foreign country compete only in the third market. Thus, these firms therefore produce only for export. This simplification appears very useful; allows one to see strategic effects of a concrete trade policy in its pure condition. In the third-market model, a domestic government can do nothing to directly hinder a foreign firm (i.e. there is no possibility for import tariffs or quotas), and the natural policy is an export subsidy, which direct effect is to help a domestic firm in a competition with its foreign rival. In the project it is supposed in model of the third market to use the two-part trade policy. The supposition is expressed, that in this model the optimum policy will appear naturally in the form of negative tariff (subsidy) at the positive payment for the license.

The sequential structure of model consists of two stages. At the first stage of government establish two-part policy $(e, \bar{v}, t)$ and $(e^*, \bar{q}, t^*)$ for interior firms. At the second stage an interior and an exterior firm simultaneously choose an output level (or export) for the third market. Using an inverse induction to find perfect subgame Nash equilibrium, Nash equilibrium at the second stage at first is considered, and then Nash equilibrium at the first stage between governments is considered, which completely realizes, that their policy will affect the value of outputs of firms at the second stage. Thus the payoff functions of firms are defined by relations (4.3), (4.4) provided that p(Q)=p*(Q).

$$\pi_i = p(Q)q_i - c_i(q_i) - t\ q_i - e\ ,\ i = 1,...N \tag{4.5}$$

$$\pi_j^* = p(Q)v_j - c_j(v_j) - t^* v_j - e^*,\ j = 1,...N^* \tag{4.6}$$

Where e, e* - payment for the license to home and foreign government accordingly; t, t*- the tariff per unit of production imposed on home and foreign firms accordingly.

We shall consider a case of homogeneous cost functions: $c_i(q) = c(q), \forall i; c_j^*(v) = c^*(v), \forall j$.

Much of the analysis of strategic profit-shifting makes use of the Cournot model of oligopolistic behavior. By stage 2, tariffs t and t* has been predetermined in stage 1 and is therefore treated as exogenous.

The first order conditions associated with maximization of (4.5), (4.6) is

$$\frac{\partial \pi_i}{\partial q_i} = p' \cdot q_i + p - c' - t = 0\ ;\ \ \frac{\partial \pi_i^*}{\partial v_j} = p' \cdot v_j + p - c^*{'} - t^* = 0\ ; \forall i, \forall j\ . \tag{4.7}$$

Because of a homogeneity $q_i = q_1, \forall i; v_j = v_1, \forall j$. The second order conditions associated with maximization of (4.5),(4.6) is

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = p'' \cdot q_i + p' + p' - c'' < 0\ , \frac{\partial^2 \pi_i^*}{\partial v_j^2} = p'' \cdot v_j + p' + p' - c^*{''} < 0\ ; \forall i, \forall j\ . \tag{4.8}$$

First order condition (4.7) makes it clear that a Cournot equilibrium is a Nash equilibrium in outputs, as (4.7) is implied by Nash condition for the case in which each player's strategy set is simply the set of possible output quantities it might produce in a one-shot simultaneous-move game. The Cournot equilibrium therefore has the same "no surprises" rationality property that any Nash equilibrium has. First order condition (4.7) could be solved in principle for the profit-maximizing choice of q_i for any given set of output choices by the other firms. This resulting implicit function is the reaction function or best-response function. The common intersection of the best-response functions (one for each firm) is the Cournot equilibrium.

An additional regularity condition that turns out to be central to the characterization of the Cournot equilibrium is the following.

$$\frac{\partial^2 \pi_i}{\partial q_i \partial v_j} = p'' \cdot q_i + p' < 0\ , \frac{\partial^2 \pi_i^*}{\partial v_j \partial q_i} = p'' \cdot v_j + p' < 0\ ; \forall i, \forall j\ . \tag{4.9}$$

This condition obviously holds for all nonconvex demand curves (including linear demand), but it can be violated if demand is very convex. Condition (4.9) is linked to many properties of the Cournot model[1].

Most importantly, condition (4.9) means that strategy variables q_i and v_j are strategic substitutes. If $\frac{\partial^2 \pi_i}{\partial q_i \partial v_j} < 0, \forall i$, this means the marginal value, $\frac{\partial \pi_i}{\partial q_i}$, of increasing firm i strategy variable decreases when the strategy variable of a rival increases.

Lemma 4.1. ***Suppose that***

1) ***the cost functions*** $c_1(q)$ ***and*** $c_1^*(q)$ ***is twice-continuously differentiable and convex, for any*** $q \geq 0$***;***
2) ***the inverse demand curve*** $p(Q)$ ***is twice-continuously differentiable and decrease, for any*** $Q \geq 0$***;***
3) ***the function*** $p(q + \widetilde{Q}) \cdot q$ ***is concave in q, for any*** $\widetilde{Q} \geq 0$. ***Then in the third-market model at two-part trade policy:***

$$\frac{\partial q_1}{\partial t} < 0, \frac{\partial q_1}{\partial t^*} > 0, \frac{\partial v_1}{\partial t} > 0, \frac{\partial v_1}{\partial t^*} < 0.$$

Corollary 4.1. $\frac{\partial v_1}{\partial t} = \alpha \frac{\partial q_1}{\partial t}$, where $\alpha \in (-1,0)$ and $\frac{\partial q_1}{\partial t^*} = \beta \frac{\partial v_1}{\partial t^*}$, where $\beta \in (-1,0)$.

As the basic model we shall consider the two-step game with complete, but imperfect information. On the first step the players 1 and 2 (the home and foreign governments) simultaneously choose their strategy and inform about them 3 and 4 - players (home and foreign firms), which on the second step simultaneously choose their strategy.

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[1] *It means that each firm's marginal revenue declines as the output of any other firm rises. It is the so-called Hahn stability condition for certain proposed dynamic adjustment mechanisms. (Note, however, that the pure Cournot model is a one-shot static game with no real-time dynamics. Any proposed dynamic adjustment is an extension to the model). Presuming that second order conditions are globally satisfied, global satisfaction of (4.9) in this context is also the Gale-Nikaido condition for uniqueness of the Cournot equilibrium. Condition (4.9) also ensures dial various comparative static properties of the model are "well-behaved".*

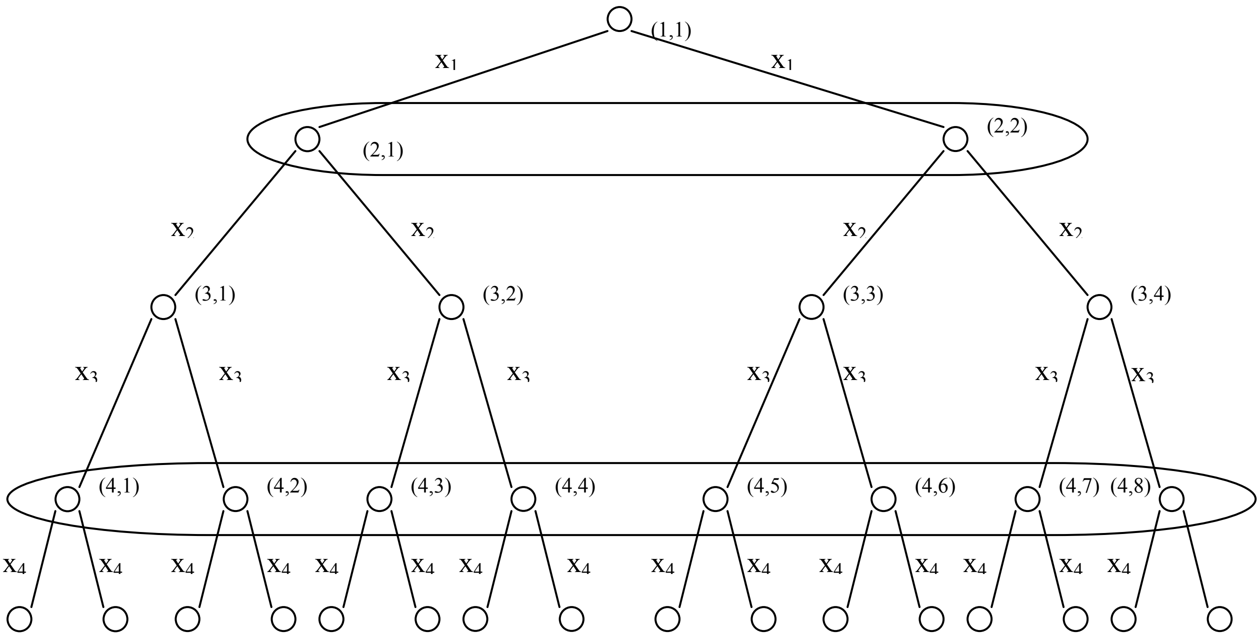
Let's designate through $x_1 = (e, \overline{v}, t) \in X_1$, $x_2 = (e^*, \overline{q}, t^*) \in X_2$ the strategies of the first and the second player, where $X_k = R_+ \times R_+ \times R$ - set of strategy k of the player (k=1,2). Let $x_3 = q_1 \in X_3$; - strategy of 3- player and $x_4 = v_4 \in X_4$ - strategy of 4 - player, where $X_k = R_+ \times R_+$ - set of strategy of k - player (k=3,4).

It $f_1(x) \equiv p \cdot q_1(t,t^*) - c(q_1(t,t^*)$; $f_2(x) \equiv p \cdot v_1(t,t^*) - c^*(v_1(t,t^*)$;

$f_3(x) \equiv \pi_1(x)$; $f_4 \equiv \pi_1^*(x)$; $x \in X = X_1 \times X_2 \times X_3 \times X_4$.

Then it is possible to define the two-step game 4 persons with complete, but imperfect information

Γ_1=< I={1,2,3,4},{ X_i }$_{i \in I}$,{ $f_i(x)$} $_{i \in I}$ >



The following existence theorem of Nash equilibrium in game Γ_1 is fair.

<u>**Theorem 4.1.**</u> ***Suppose that***

1) the cost functions $c_1(q)$ ***and*** $c_1^*(q)$ ***is twice-continuously differentiable increasing and convex, for any*** $q \geq 0$***;***

2) the inverse demand curve $p(Q)$ ***is twice-continuously differentiable and decrease, for any*** $Q \geq 0$ ***;***

3) ***the function*** $p(q + \widetilde{Q}) \cdot q$ ***is concave in q, for any*** $\widetilde{Q} \geq 0$**.**

4) $\exists \hat{Q}$, *that* $p(Q) = 0, \forall Q \geq \hat{Q}$,

then in game Γ_1 exists perfect subgame Nash equilibrium.

From the given theorem we get a remarkable corollary.

<u>Corollary 4.2.</u> In the third-market model optimum two-part tariff t which maximizies the government's incomes is negative (subsidy) and the optimum payment for the license is equal to flowing sales proceeds of a internal firm.

<u>Example 4.1.</u> (model of the third market). One market, two governments, two firms. The inverse function of demand $p(Q) = a - b \cdot Q$. The costs functions of home and foreign firms: $C(q) = cq$; $C^*(q) = c^* q$.

For searching the optimum two-part trade policy we apply a method of an inverse induction. Thus we consider various combinations of applying or not applying the two-part trade policies by different governments. The outcomes are given in Tab.4.1

From the given example it is visible, that the optimum two-part trade policy is the subsidy at the positive payment for the license.

If the two-part trade policy is applied only by one government, it is also necessary to notice, that the optimum subsidy is increased because of advantage in relative costs of interior firm. Firms, which "need" help at competition with exterior firms represent less attractive objects for subsidizing from the point of view of government. Besides we see that in this case subsidy forces interior firm to more aggressive function of the best answer, that in turn forces a foreign firm to produce less. The optimum interior subsidy moves a firm on a leading Stakelberg output level, and a foreign subsidy makes at a level of the follower. The government can convert that advantage (that it applies the solution first) into equivalent advantage of an interior firm. The interior firm has stimulus for acceptance of a primary solution, which changes strategic interaction between firms.

If the two-part policy is applied by both governments (at a maximization G_k), their optimum policy still will be the subsidy at the positive tariff. However in case of a maximization of welfare ($G_k + \pi_k$) the game at a level of governments looks like a prisoners` dilemma made, as both countries are in the worse position in a condition of a strategic equilibrium, than at free trade, however each has stimulus for a deviation from free trade.

Let's consider for an example 4.1, whether the optimum two-part trade policy of the Pareto-optimal with respect to the policies of each government (G+G *), i.e. the case when between governments is present collusion. In this case we have the following first-best outcome:

$t+t^* = (a-c)/2 > 0$, which rather differs from the individually tariffs $t=t^* =- (a-c /5 < 0$. For example, if $t=t^* = (a-c)/4$, $G=G^*=(a-c)^2/8b>2(a-c)^2/25b$; $\pi= \pi^*=0$; $q=q^* = (a-c)/4b$; $e=e^*=(a-c)^2/16b$. Here, the game also has the

prisoners' dilemma flavor in the sense that collusion between the two governments would have resulted in greater revenue for each (assuming equal division of total revenue) that individually rational policies.

4.2 Two-part trade policy in model of the reciprocal markets with homogeneous structure of cost function.

In this case $c_i\,(q_i) = c(q_i), \forall i$ and $c_j^*(q_j) = c^*(q_j^*), \forall j$.

The economic result of the firms' activity is determined by the consequent functions of profit:

$$\pi_i = p(Q)q_i - c_i(q_i + q_i^*) + p^*(Q^*)q_i^* - t^*q_i^* - e^*,\ i = 1,...N$$

$$\pi_j^* = p^*(Q^*)v_j^* - c_j^*(v_j + v_j^*) + p(Q)v_j - tv_j - e,\ j = 1,...N^* \qquad (4.18)$$

Where e, e* - payment for the license to home and foreign government accordingly; t, t*- the tariff per unit of production imposed on home and foreign firms accordingly.

Lemma 4.2 : *If*

1) $c_i(q) \in C^2, c'(q) > 0, c''(q) > 0, q > 0$; $c^*(v) \in C^2, (c^*(v))' > 0, (c^*(v))'' > 0, v > 0$;

2) $p(Q), p^*(Q^*) \in C^2$; $p'(Q) < 0, Q \geq 0$, $p^{*\prime}(Q^*) < 0, Q^* \geq 0$;

3) $p'(Q) + p''(Q)\cdot q_i < 0, \forall(i, Q \geq 0, q_i \geq 0)$; $p^{*\prime}(Q) + p^{*\prime\prime}(Q)\cdot v_j < 0, \forall(j, Q^* \geq 0, v_j \geq 0)$;

4) $p'(Q) - c''(q_i) < 0, \forall(i, Q \geq 0, q_i \geq 0)$; $p^{*\prime}(Q) - c^{*\prime\prime}(q_j) < 0, \forall(j, Q^* \geq 0, v_j \geq 0)$; $i = 1,...N$; $j = 1,...N$,

then in the model of the reciprocal markets under the two-part trade policy the condition are executed:

$$\frac{\partial q_i}{\partial t} > 0, \frac{\partial v_j}{\partial t} < 0; \qquad \frac{\partial v_j^*}{\partial t^*} > 0, \frac{\partial q_i^*}{\partial t^*} < 0;$$
$$\frac{\partial q_i^*}{\partial t} < 0, \frac{\partial v_j^*}{\partial t} > 0; \qquad \frac{\partial v_j}{\partial t^*} < 0, \frac{\partial q_i}{\partial t^*} > 0; \qquad \textit{for } \forall i = 1,...N; \forall j = 1,...N$$

The importance of this Lemma 4.2. is that for the reciprocal-markets model it establishes a very important feature: « the import tariffs of the two-part trade policy reduce the level of internal sales of external firms and increase domestic sales of internal firms ». Though the given fact is known, but for the model under consideration it is new.

From the proved Lemma 4.2. it is also easy to get:

Corollary 4.3. $\dfrac{\partial q_1}{\partial t} = \alpha \dfrac{\partial v_1}{\partial t}$, *where* $\alpha \in (-1,0)$ *and* $\dfrac{\partial v_1^*}{\partial t^*} = \beta \dfrac{\partial q_1^*}{\partial t^*}$, *where* $\beta \in (-1,0)$.

4.2.1 Policy of a maximization of the governmental incomes.

The purposes of government are defined by functions.

$$G(t,t^*) = \sum_{j=1}^{N^*} \{pv_j(t,t^*) - c^*(v_j(t,t^*))\} \text{- income of home government} \qquad (4.25)$$

$$G^*(t,t^*) = \sum_{i=1}^{N} \{p^* q_i^*(t,t^*) - c(q_i^*(t,t^*))\} \text{- income of foreign government} \qquad (4.26)\ .$$

The given type of government we shall called G- type. For the governments of G-type the level of optimum two-part trade policy is defined by the conditions of equality to zero of the income from sales of foreign firms.

As the basic model we shall consider the two-step game with complete, but imperfect information. On the first step the players 1 and 2 (the home and foreign governments) simultaneously choose their strategy and inform about them N+N* - players (N-home and N*- foreign firms), which on the second step simultaneously choose their strategy.

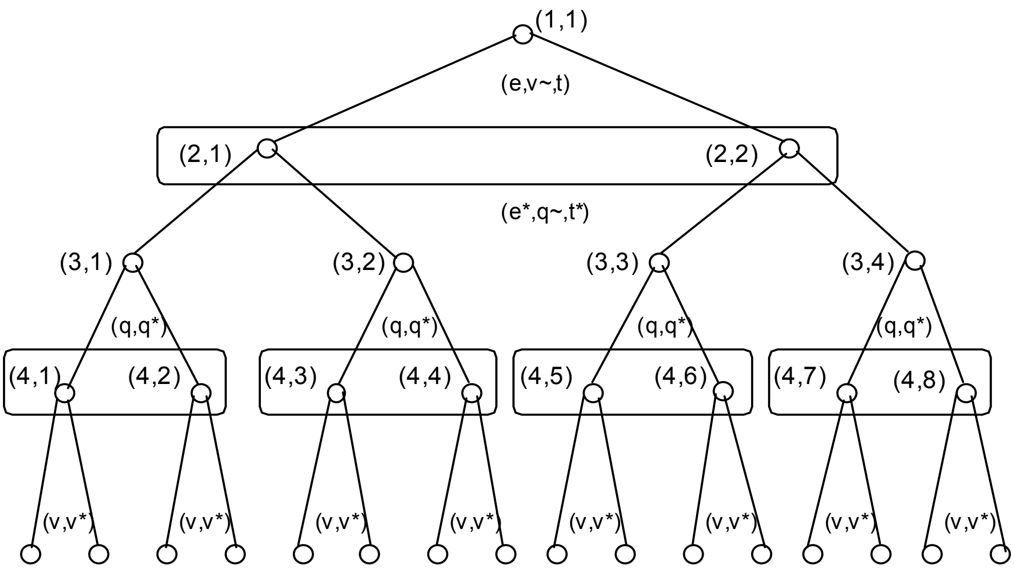
Let's designate through $x_1 = (e, \widetilde{v}, t) \in X_1$, $x_2 = (e^*, \widetilde{q}, t^*) \in X_2$ the strategies of the first and the second player, where $X_k = R_+ \times R_+ \times R$ - set of strategy k of the player (k=1,2). Let $x_i = (q_i, q_i^*) \in X_i$; - strategy of i- player $(i = 3,4 \ldots N+2)$, and $x_j = (v_j, v_j^*) \in X_j$ - strategy of j - player $(j = N+3, \ldots N + N^* + 2)$, where $X_k = R_+ \times R_+$ - set of strategy of k - player (k=3,…N+N*+2).

It $f_1(x) \equiv G(x)$; $f_2(x) \equiv G^*(x)$; $f_i(x) \equiv \pi_i(x)$, $i = 3, \ldots N+2$;

$f_j \equiv \pi^*_{j-2-N}(x)$, $j = N+3, \ldots (N + N^* + 2)$; $x \in X = X_1 \times X_2 \times \ldots \times X_{N+N^*+2}$

Then it is possible to define the two-step game N+N * + 2 persons with complete, but imperfect information

$$\Gamma = \langle I = \{1,2 \ldots N+N^*+2\}, \{X_i\}_{i \in I}, \{f_i(x)\}_{i \in I} \rangle$$



The following existence theorem of Nash equilibrium in game Γ is fair.

<u>Theorem 4.2.</u> ***If***

1) $c_i(q) \in C^2, c'(q) > 0, c''(q) > 0, q > 0$ ***;*** $c^*(v) \in C^2, (c^*(v))' > 0, (c^*(v))'' > 0, v > 0$ ***;***

2) $p(Q), p^*(Q^*) \in C^2$ ***;*** $p'(Q) < 0, Q \geq 0$, $p^{*\prime}(Q^*) < 0, Q^* \geq 0$ ***;***

3) $p'(Q) + p''(Q) \cdot q_i < 0, \forall (i, Q, q_i)$ ***;*** $p^{*\prime}(Q) + p^{*\prime\prime}(Q) \cdot v_j < 0, \forall (j, Q^*, v_j)$ ***;***

4) $p'(Q) - c''(q_i) < 0, \forall (i, Q, q_i)$ ***;*** $p^{*\prime}(Q) - c^{*\prime\prime}(q_j) < 0, \forall (j, Q^*, v_j)$ ***;***

5) $\exists \tilde{Q}$, *that* $p(Q) = 0, \forall Q \geq \tilde{Q}$ ***;*** $\exists \tilde{Q}^*$, *that* $p^*(Q^*) = 0, \forall Q^* \geq \tilde{Q}^*$ ***,***

($i = 3, \ldots N + 2$ ***;*** $j = N + 3, \ldots N + N^* + 2$ ***), then in game Γ exists perfect subgame Nash equilibrium.***

From the given theorem we get a remarkable corollary.

<u>Corollary 4.4.</u> In the homogeneous case for N=N * = 1 optimum two-part tariff t which maximizes the government's incomes is negative (subsidy) and the optimum payment for the license equals to the current of a foreign firm.

Let's consider nontrivial example of the two-part optimum trade policy.

<u>Example 4.2.</u> Two markets, two governments, two firms.

The inverse functions of demand in the home and foreign market p (Q) =1-Q are known; p * (Q *) = 1-Q*.

Functions of costs of home and foreign firms: C (q) =cq2/2; C * (q) =cq2/2.

The functions of firms' profit and the functions of the income of governments are defined under the formulas (4.18), (4.25) and (4.26).

It could be readily verified that in this modelling example the conditions of Lemma 4.2 and Theorem 4.2 are carried out.

So the optimum out puts of firms depending on a level of two-part tariff:

$$q_1(t,t^*) = \frac{1}{3}\cdot\frac{4c^2t^*+6c+3t+5ct^*+4ct+2c^2t+3}{(2c+3)(2c+1)};\, q_1^*(t,t^*) = -\frac{1}{3}\cdot\frac{4c^2t^*-6c+6t^*+11ct^*+4ct+2c^2t-3}{(2c+3)(2c+1)}$$

$$v_1^*(t,t^*) = -\frac{1}{3}\cdot\frac{4c^2t-6c+6t+11ct+4ct^*+2c^2t^*-3}{(2c+3)(2c+1)};\, v_1^*(t,t^*) = \frac{1}{3}\cdot\frac{4c^2t+6c+3t^*+5ct+4ct^*+2c^2t^*+3}{(2c+3)(2c+1)}$$

Thus of a payoff function of governments:

$$G(t,t^*) = -\frac{1}{18}\cdot(2c^2t^*+4c^2t+4ct^*-6c+6t+11ct-3)\frac{(18c^2+21c+6+6t-2ct^*+20ct+15c^2t+2c^3t^*+4c^3t)}{(2c+3)^2(2c+1)^2}$$

$$G^*(t,t^*) = -\frac{1}{18}\cdot(2c^2t+4c^2t^*+4ct-6c+6t^*+11ct^*-3)\frac{(18c^2+21c+6+6t^*-2ct+20ct^*+15c^2t^*+2c^3t+4c^3t^*)}{(2c+3)^2(2c+1)^2}$$

Functions are continuous and concave by t and t* accordingly.

Thus the optimum tariff:

$$t = -\frac{13c^2+4c^3+3+10c}{(2+c)(4c^3+13c^2+17c+6)} < 0; \quad t^* = -\frac{13c^2+4c^3+3+10c}{(2+c)(4c^3+13c^2+17c+6)} < 0.$$

Thus optimum output of firms (at this level the quotas $\overline{q} = q_1^*; \overline{v} = v_1$) are also established:

$$q_1 = \frac{(c+3)(2c+1)}{(2+c)(4c^3+13c^2+17c+6)};\, q_1^* = \frac{(c+1)(4c+3)}{(4c^3+13c^2+17c+6)};\, v_1^* = \frac{(c+1)(4c+3)}{(4c^3+13c^2+17c+6)};\, v_1^* = \frac{(c+3)(2c+1)}{(2+c)(4c^3+13c^2+17c+6)}$$

The optimum payment for the license:

$$e = p^*\cdot q^* - C(q^*) - t\cdot q^* = \frac{1}{2}\cdot\frac{(c+1)(4c+3)(27c^3 61c+46c+12+4c^4)}{(4c^3+13c^2+17c+6)^2(2+c)} > 0;$$

$$e^* = p\cdot v - C^*(v) - t\cdot v = \frac{1}{2}\cdot\frac{(c+1)(4c+3)(27c^3 61c+46c+12+4c^4)}{(4c^3+13c^2+17c+6)^2(2+c)} > 0\cdot$$

The example shows, that the optimum two-part policy in case of two countries, two governments and two firms is the subsidy at the positive payment for the license. Thus such policy encourages firms to export greater than interior output level.

4.2.2 Policy of a maximization of the domestic welfare.

The purposes of government are defined by functions.

$$W(t,t^*) = \int_0^{Q(t,t^*)} p(s)ds - \sum_{i=1}^{N} c(q_i(t,t^*)) - \sum_{j=1}^{N^*} c^*(v_j(t,t^*)) \quad (4.31)$$ - domestic welfare of a home country.

$$W^*(t,t^*) = \int_0^{Q^*(t,t^*)} p^*(s)ds - \sum_{i=1}^{N} c(q_i^*(t,t^*)) - \sum_{j=1}^{N^*} c^*(v_j^*(t,t^*)) \quad (4.32)$$ - domestic welfare of a foreign country.

The given type of government we shall called W- type.

In this case a government maximizes domestic welfare consisting of the sum of an excess of the customer, profit of an internal firm and it's own incomes from the tax. For the governments of W – type the level of optimum two-part trade policy is defined by the conditions of equality to zero of the income from sales of foreign firms.

As the basic model we shall consider the two-step game with complete, but imperfect information. On the first step the players 1 and 2 (the home and foreign governments) simultaneously choose their strategy and inform about them N+N* - players (N-home and N*- foreign firms), which on the second step simultaneously choose their strategy.

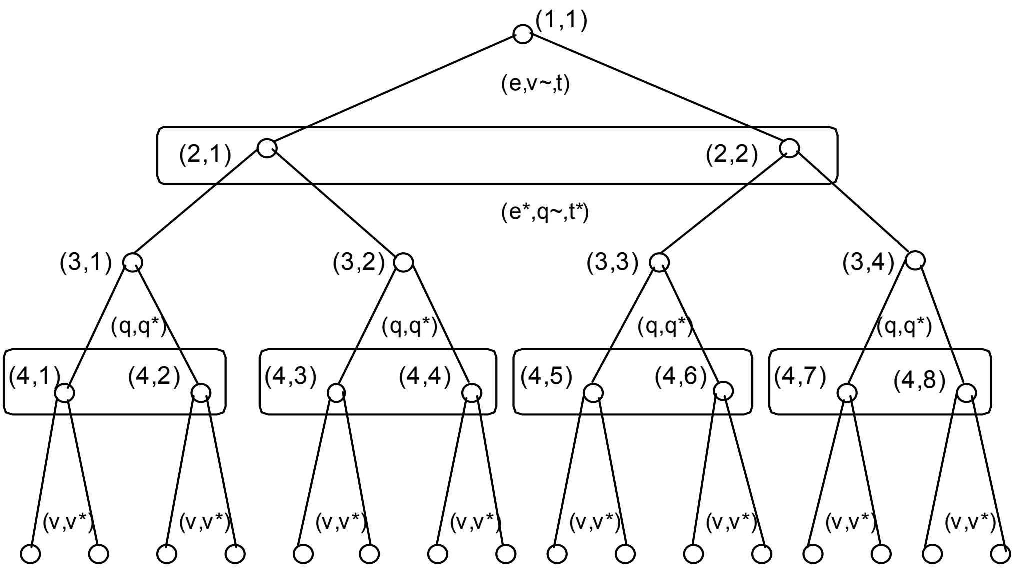
Let's designate through $x_1 = (e, \widetilde{v}, t) \in X_1$, $x_2 = (e^*, \widetilde{q}, t^*) \in X_2$ the strategies of the first and the second player, where $X_k = R_+ \times R_+ \times R$ - set of strategy k of the player (k=1,2). Let $x_i = (q_i, q_i^*) \in X_i$; - strategy of i- player $(i = 3,4...N+2)$, and $x_j = (v_j, v_j^*) \in X_j$ - strategy of j - player $(j = N+3,...N+N^*+2)$, where $X_k = R_+ \times R_+$ - set of strategy of k - player (k=3,…N+N*+2).

It $h_1(x) \equiv W(x)$; $h_2(x) \equiv W^*(x)$; $h_i(x) \equiv \pi_{i-2}(x)$, $i = 3,...N+2$;

$h_j \equiv \pi^*_{j-2-N}(x)$, $j = N+3,...(N+N^*+2)$; $x \in X = X_1 \times X_2 \times ... \times X_{N+N^*+2}$

Then it is possible to define the two-step game N+N * + 2 persons with complete, but imperfect information

$$\boldsymbol{\Gamma^W = \langle I=\{1,2\ldots N+N^*+2\}, \{X_i\}_{i\in I}, \{h_i(x)\}_{i\in I} \rangle}$$



The following existence theorem of Nash equilibrium in game Γ is fair.

__**Theorem 4.3 :**__ ***If***

1) $c\ (q) \in C^2, c'(q) > 0, c''(q) > 0, q > 0$ ***;*** $c^*(v) \in C^2, (c^*(v))' > 0, (c^*(v))'' > 0, v > 0$ ***;***

2) $p(Q), p^*(Q^*) \in C^2$ ***;*** $p'(Q) < 0, Q \geq 0$***,*** $p^{*\prime}(Q^*) < 0, Q^* \geq 0$ ***;***

3) $p'(Q) + p''(Q) \cdot q_i < 0, \forall (i, Q, q_i)$ ***;*** $p^{*\prime}(Q) + p^{*\prime\prime}(Q) \cdot v_j < 0, \forall (j, Q^*, v_j)$ ***;***

4) $p'(Q) - c''(q_i) < 0, \forall (i, Q, q_i)$ ***;*** $p^{*\prime}(Q) - c^{*\prime\prime}(q_j) < 0, \forall (j, Q^*, v_j)$ ***;***

5) $\exists \tilde{Q}$, *что* $p(Q) = 0, \forall Q \geq \tilde{Q}$ ***;*** $\exists \tilde{Q}^*$, *что* $p^*(Q^*) = 0, \forall Q^* \geq \tilde{Q}^*$***,***

($i = 3, \ldots N + 2$ ***;*** $j = N + 3, \ldots N + N^* + 2$ ***), then in game Γ exists perfect subgame Nash equilibrium.***

Proof is presented in the Appendix.

Let's consider nontrivial examples of the two-part optimum trade policy.

Now we shall consider examples, which confirm expressed before a hypothesis.

We make the comparative analysis of outcomes of examples 4.3, 4.4, 4.6, 4.7 (See. Application) and before the obtained outcomes Fuerst and Kim (1997). All outcomes we shall note in the table 4.2:

Tab. 4.2

			Melnik V.N.				Fuerst & Kim			
N*	N	c	Z_G	Z_G*	Z_W	Z_W*	Z_G	Z_G*	Z_W	Z_W*
1	1	low	subsidy	subsidy	subsidy	subsidy	subsidy	-[2]	subsidy	-
1	1	high	subsidy	subsidy	subsidy	subsidy	subsidy	-	subsidy	-
2	1	low	subsidy	subsidy	subsidy	subsidy	tariff =0	-	subsidy	-
2	1	high	subsidy	subsidy	subsidy	tariff >0	tariff =0	-	subsidy	-
≥3	1	low	tariff >0	subsidy	subsidy	subsidy	tariff >0	-	subsidy	-
≥3	1	high	subsidy	subsidy	subsidy	tariff >0	tariff >0	-	subsidy	-
2	2	low	subsidy	subsidy	subsidy	subsidy	-	-	-	-
2	2	high	subsidy	subsidy	subsidy	subsidy	-	-	-	-
3	3	low	subsidy	subsidy	subsidy	subsidy	-	-	-	-
3	3	high	subsidy	subsidy	subsidy	subsidy	-	-	-	-

From the table it is visible that for the considered examples[3]

1) A subsidy appear (both for G-government and for W-government) more often and under other conditions, than in Fuerst and Kim(1997) models and it depends on a difference among foreign and home firms and from a cost value.

2) For G-government, when all firms have high costs, independently of a number of firms in the market the optimal policy is a subsidy;

3) At a internal government (in case of G) the optimal policy appear by the way of a positive tariff, when all firms have low costs, and a number of external firms is more internal firms on 2 or more. Thus the optimal policy of a external government will be always a subsidy;

4) At a internal government (in case of W) the optimal policy will be always a subsidy, if it is more internal firms in than the market, than external firms. Thus the optimal policy of a external government will be a subsidy at low costs and a positive tariff at high costs. In case of equality of a number of firms in the market the optimal policy is always a subsidy.

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[2] Here label "-" means, that this case was not analysis (Fuerst & Kim(1997))

[3] Generally results to receive very difficultly

In examples 4.3 and 4.4 with increase of parameter "c" the value of the governmental income in the beginning will increase at the expense of the greater value of the subsidy (these costs of government compensate by a high positive payment for the license), and then the income decreases as the costs of firms will increase so that the governments can not compensate costs on subsidizing by a payment for the license. In this case all the firms have identical cost structure, a revenue-maximizing government uses its two-part strategy to encourage the exterior firms to behave as monolithic Stackelberg leader in the second stage of game. This implies that the revenue-maximizing per unit of fee is either negative (when the cost functions of firms are high, at any amount of firms), or positive (when the cost functions of firms are small, and number of exterior firms large). These stimulus are modified in case of government W that also is concerned with consumer surplus. In particular welfare-maximizing government will always choose a negative per unit fee (a subsidy) to stimulate total production. This situation is possible at a high payment for the license. Thus if the cost functions of firms are high (interior and exterior) then per unit of fee of interior firm is positive (the interior firms do not buy the license).

In addition for example 4.6 the case is considered, whether the optimum two-part trade policy of the Pareto-optimal concerning summarized welfare (W+W *), i.e. the case when between governments is present collusion.

In this case we have the following first-best outcome:

$t=t^*=-0.2857<0$ which a little bit differs from the individually optimal tariffs $t=t^*=-0.3925<0$(in this case $W=W^*=0.3552$) . Besides in this case when between governments is present collusion $W=W^*=0.35715>0.3552$; $q=0.1429$, $q^*=0.4286$; $v=0.4286$, $v^*=0.1429$; $e=e^*=0.2602$. Here, the game also has the prisoners' dilemma flavor in the sense that collusion between the two governments would have resulted in greater summarized welfare for each that individually rational policies.

4.3 Two-part trade policy in model of the reciprocal markets with inhomogeneous structure of cost function.

In this case all the functions of costs are different and we can not reduce the tariff at the expense of increasing the payment for the license, because the firms with high costs can fall out of the market if the payment for the license is high.

If n and n* are extreme firms, that is the firms with numbers n+1 and (n* +1) have negative profit at the optimum two-part trade policy $z=(e,\overline{v},t)$, $z^*=(e^*,\overline{q},t^*)$.

4.3.1 Policy of a maximization of the governmental incomes.

In this case payoff functions of governments:

$$G_{n^*}(t,t^*) = t(I_{n^*}(t,t^*) - n^* v_{n^*}(t,t^*)) + n^*(p v_{n^*}(t,t^*) - c^*_{n^*}(v_{n^*}(t,t^*))) \qquad (4.38)$$

$$G^*_n(t,t^*) = t^*(I^*_n(t,t^*) - n q^*_n(t,t^*)) + n(p^* q^*_n(t,t^*) - c_n(q^*_n(t,t^*))) \qquad (4.39)$$

where I_{n^*}, I^*_n - level of import in a home and a foreign country accordingly. The optimum policy is defined by optimization on t (t *) and n (n *). In this model the optimum policy of government will also depend on a degree of convexity and a degree of heterogeneity of functions of costs.

The example shows, that the optimum two-part policy in case of two countries, two governments and two firms is the subsidy at the positive payment for the license. Thus such a policy encourages firms to export greater than interior output level.

Further we shall consider examples for an inhomogeneous case.

4.3.2 Policy of a maximization of the domestic welfare.

In this case payoff functions of governments:

$$W_{n^*}(t,t^*) = \int_0^{\sum_{i=1}^{N} q_i(t,t^*) + I_{n^*}(t,t^*)} p(s)ds - (p-t)\cdot I_{n^*}(t,t^*) + n^*\cdot(v_{n^*}(t,t^*)\cdot(p-t) - c^*_{n^*}(v_{n^*}(t,t^*)) - \sum_{i=1}^{N} c_i(q_i(t,t^*)) \qquad (4.40)$$

$$W^*_n(t,t^*) = \int_0^{\sum_{j=1}^{N^*} v_j(t,t^*) + I^*_n(t,t^*)} p^*(s)ds - (p^*-t^*)\cdot I^*_{n^*}(t,t^*) + n\cdot(q^*_n(t,t^*)\cdot(p^*-t^*) - c_n(q^*_n(t,t^*)) - \sum_{j=1}^{N^*} c^*_j(v^*_j(t,t^*)) \qquad (4.41)$$

where I_{n^*}, I^*_n - level of import in a home and a foreign country accordingly. The optimum policy is defined by optimization on t (t *) and n (n *). In this model the optimum policy of government will also depend on a degree of convexity and a degree of heterogeneity of functions of costs.

The example shows, that the optimum two-part policy in case of two countries, two governments and two firms is the subsidy at the positive payment for the license. Thus such policy encourages firms to export greater than interior output level.

Conducted in examples 4.8-4.11 analysis allows to state more deep and new outcomes (than Fuerst &Kim(1997)) for an inhomogeneous case. Besides now it is possible to conduct the comparative analysis of outcomes examples 4.8-4.11 (see application). All outcomes we shall tabulate 4.4:

Tab. 4.4

		Melnik V.N.					Fuerst & Kim				
N	Heterogeneity	N*	Z_G	Z_G*	Z_W	Z_W*	N*	Z_G	Z_G*	Z_W	Z_W*
1	Low	3	subsidy	subsidy	subsidy	subsidy	3	tariff ≥0	-[4]	tariff ≥0	-
1	middle	2	subsidy	subsidy	subsidy	subsidy	2	tariff ≥0	-	tariff ≥0	-
1	high	1	subsidy	subsidy	tariff ≥0	subsidy	1	subsidy	-	subsidy	-

Here **parameter N* - endogenous**

From the table it is visible that for the considered examples[5]

1) For G-government at any degree of a heterogeneity the internal government will establish the high payment of an entrance for external firms before, than its the policy will be replaced from a subsidy with a positive tariff, i.e. the optimal policy will be always a subsidy.

1) For W-government the situation is similar, except for a case when a number of external and internal firms is equal to one, and the costs of internal firms exceed costs of external firms, then the optimal policy of internal government is a positive tariff.

5. Modified two-part trade policy.

In the work of Melnik (2000) for the simpler models it was offered to enter parameter τ- a tariff for the local producer in hope, that in oligopoly case it will result in the subsidy. For a government such as G- type the tariff turned to be positive. For a government such as W-type has really resulted in the subsidy (τ< 0). Moreover, there will be a great number of the two-part policies, as for the optimum tariffs a condition must be observed: $t + \tau = -(1 - c)$, that gives a home government additional tools of regulating.

[4] Here label "-" means, that this case was not analysis (Fuerst & Kim(1997))
[5] Generally results to receive very difficultly

We shall use this effect. It is offered to apply the two-part trade policy not only to foreign, but also to local firms $(\tilde{e},\tilde{v},\tau)$, $(\tilde{e}^*,\tilde{q},\tau^*)$. The modified two-part trade policy, thus is defined $\tilde{z}=(e,\bar{v},t,\tilde{e},\tilde{v},\tau)$, $\tilde{z}^*=(e^*,\bar{q},t^*,\tilde{e}^*,\tilde{q},\tau^*)$.

Thus the given trade policy generalizes many existing trade policies:

1) $\tilde{z}_0=(0,\infty,0,0,\infty,0)$ - free trade; 2) $\tilde{z}_1=(0,\infty,t,0,\infty,0)$ - simple tariff; 3) $\tilde{z}_2=(e,\bar{v},0,0,\infty,0)$ - simple quota; 4) $\tilde{z}_3=(e,\bar{v},t,0,\infty,0)$ - two-part trade policy on a foreign firm; 5) $\tilde{z}_4=(0,\infty,0,0,\infty,\tau),\tau<0$ - subsidizing of export; 6) $\tilde{z}_5=(0,\infty,0,\tilde{e},\tilde{v},0)$ - voluntary export restriction; 7) $\tilde{z}_6=(0,\infty,0,\tilde{e},\tilde{v},\tau)$ - two-part trade policy on home firm; 8) $\tilde{z}_7=(0,\infty,0,\tilde{e},\tilde{v},\tau),\tau<0,\tilde{e}>0$ - export credit subsidies.

Thus the payoff functions of firms are changed:

$$\tilde{\pi}_i=p(Q)q_i-c\ (q_i+q_i^*)+p^*(Q^*)q_i^*-t^*q_i^*-e^*-\tau q_i-\tilde{e}\,,\ i=1,...N \qquad (5.1)$$

$$\tilde{\pi}_j^*=p^*(Q^*)v_j^*-c^*(v_j+v_j^*)+p(Q)v_j-tv_j-e-\tau^*v_j^*-\tilde{e}^*,\ i=1,...N^* \qquad (5.2)$$

5.1. Policy of a maximization of the governmental incomes.

In this case payoff functions of governments:

$$\tilde{G}=\sum_{j=1}^{N^*}\left\{pv_j(t)-c^*(v_i(t))\right\}+\sum_{i=1}^{N}\left\{pq_i(\tau)-c(q_i(\tau))\right\} \text{ - income of home government} \qquad (5.3)$$

$$\tilde{G}^*=\sum_{i=1}^{N}\left\{p^*q_i^*(t^*)-c(q_i^*(t^*))\right\}+\sum_{j=1}^{N^*}\left\{p^*v_j^*(\tau^*)-c^*(v_j^*(\tau^*))\right\} \text{ - income of home government} \qquad (5.4)$$

The hypothesis is advanced, that in case a government of a maximizes its income the modified two-part trade policy will not be the subsidy at any conditions. These ideas are best illustrated in the context of concrete examples.

Example 5.1.1 Let's assume are known inverse demand function in the home and foreign market and cost functions of firms:

$$p(Q)=a_1-b_1\cdot Q\,,\ Q=q_1+q_2;\ \ p(Q^*)=a_2-b_2\cdot Q^*,\ Q^*=q_1^*+q_2^*;\ \ a_1>c,a_2>c,b_1>0,b_2>0$$

$$C(q)=c\cdot q+d\,;\ \ C^*(q)=c\cdot q+d\,.$$

In this case optimal modified two-part trade policy is defined by conditions:

$$t+t_1=\frac{a_1-c}{2}>0; \qquad t^*+t_1^*=\frac{a_2-c}{2}>0; \qquad (5.5)$$

$$q_1 = \frac{a_1 - c}{2b_1} - \frac{t_1}{b_1};\ q_1^* = \frac{a_2 - c}{2b_2} - \frac{t^*}{b_2};\ q_2 = \frac{a_1 - c}{2b_1} - \frac{t}{b_1};\ q_2^* = \frac{a_2 - c}{2b_2} - \frac{t_1^*}{b_2}. \qquad (5.6)$$

$$G = \frac{(a_1 - c)^2 - 8d \cdot b_1}{4b_1};\ G^* = \frac{(a_2 - c)^2 - 8d \cdot b_2}{4b_2}.$$

<u>Proposition 5.1.1.</u> In case of linear cost functions and linear inverse functions of demand, there is an infinitely many optimal modified two-part trade policy maximizing the governmental incomes. Thus the case is possible when the interior production level are not equal to zero and the case of the subsidy is impossible.

<u>Proof.</u> As we have infinite number of solutions of the equations (5.5), there is an infinite number optimal modified two-part trade policy maximizing the governmental incomes. Let t < 0 (subsidy), then from (5.5) $t_1 > \frac{a_1 - c}{2}$, and froma (5.6) $q_1 = \frac{a_1 - c}{2b_1} - t_1 < 0$. But the production level can not be negative, then $t \geq 0$, i.e. the subsidy is impossible.

<u>Example 5.1.2</u> We shall assume, that in conditions of an example 5.1.1 there is epy collusion between governments which maximize in common summarized income ($G + G^*$). Besides let $a_1 = a_2, b_1 = b_2$. In this case we obtain outcome completely conterminous with outcome of an example 5.1.1. In this case: $t + t_1 = \frac{a-c}{2} > 0$;

$$t^* + t_1^* = \frac{a-c}{2} > 0;\ q_1 = \frac{a-c}{2b} - \frac{t}{b};\ q_1^* = \frac{a-c}{2b} - \frac{t^*}{b};\ q_2 = \frac{a-c}{2b} - \frac{t}{b};\ q_2^* = \frac{a-c}{2b} - \frac{t_1^*}{b};$$

$$G = \frac{(a-c)^2 - 8d \cdot b}{4b};\ G^* = \frac{(a-c)^2 - 8d \cdot b}{4b}.$$

Thus it is fair:

<u>Proposition 5.1.2.</u> In case of identical cost functions and identical linear inverse functions of demand, the modified two-part trade policy maximizing the governmental incomes gives the Pareto optimum outcome concerning the joint income.

<u>Corollary.</u> In case of linear cost functions and linear inverse functions of demand optimal modified, two-part trade policy maximizing the governmental incomes dominates on other trade policy instruments.

<u>5.2. Policy of a maximization of the domestic welfare.</u>

The purposes of government are defined by functions.

$$W(t,t^*) = \int_0^{Q(t,t^*)} p(s)ds - \sum_{i=1}^{N} c(q_i(t,t^*)) - \sum_{j=1}^{N^*} c^*(v_j(t,t^*))$$ (5.7) - domestic welfare of a home country.

$$W^*(t,t^*) = \int_0^{Q^*(t,t^*)} p^*(s)ds - \sum_{i=1}^{N} c(q_i^*(t,t^*)) - \sum_{j=1}^{N^*} c^*(v_j^*(t,t^*))$$ (5.8) - domestic welfare of a foreign country.

In case of a maximization of domestic welfare the modified policy is the subsidy.

Example 5.2.1 Let's assume are known inverse demand function in the home and foreign market and cost functions of firms:

$p(Q) = a_1 - b_1 \cdot Q$, $Q = q_1 + q_2$; $p(Q^*) = a_2 - b_2 \cdot Q^*$, $Q^* = q_1^* + q_2^*$; $a_1 > c, a_2 > c, b_1 > 0, b_2 > 0$

$C(q) = c \cdot q + d$; $C^*(q) = c \cdot q + d$.

In this case optimal modified two-part trade policy is defined by conditions:

$$t + t_1 = -(a_1 - c) < 0;\ t^* + t_1^* = -(a_2 - c) < 0; \tag{5.9}$$

$$q_1 = -\frac{t_1}{b_1};\ q_1^* = -\frac{t^*}{b_2};\ q_2 = -\frac{t}{b_1};\ q_2^* = -\frac{t_1^*}{b_2}. \tag{5.10}$$

$$W = \frac{(a_1 - c)^2 - 4d \cdot b_1}{2b_1};\ W^* = \frac{(a_2 - c)^2 - 4d \cdot b_2}{2b_2}.$$

Proposition 5.2.1 In case of linear cost functions and linear inverse functions of demand there is an infinitely many optimal modified two-part trade policy (as the subsidy) maximizing the domestic welfare. Thus the case is possible when the interior production level represents the competitive outcome.

Proof. As we have infinitely many of solutions of the equations (5.9), there is an infinitely many optimal modified two-part trade policy maximizing the domestic welfare.

Let t > 0, then from (5.10) $q_2 < 0$. But production can not be negative, then $t \le 0$. Analogously $t_1 \le 0$, $t^* \le 0$, $t_1^* \le 0$. From (5.9) follows that t, t_1 and t^*, t_1^* not equal to zero simultaneously, i.e. optimal modified two-part trade policy is subsidy.

Example 5.2.2 We shall assume, that in conditions of an example 5.2.1 there is epy collusion between governments which maximize common the domestic welfare ($W + W^*$). Besides let $a_1 = a_2, b_1 = b_2$. In this case we obtain outcome completely conterminous with outcome of an example 5.2.1. In this case: $t+t_1=-(a-c)<0$;

$t^*+t_1^*=-(a-c)<0$; $q_1=-\frac{t_1}{b}$; $q_1^*=-\frac{t^*}{b}$; $q_2=-\frac{t}{b}$; $q_2^*=-\frac{t_1^*}{b}$

$W=\frac{(a-c)^2-4d\cdot b}{2b}$; $W^*=\frac{(a-c)^2-4d\cdot b}{2b}$.

Thus it is fair:

Proposition 5.2.2. In case of identical cost functions and identical linear inverse functions of demand, the modified two-part trade policy maximizing the domestic welfare gives the Pareto optimum outcome concerning the joint domestic welfare.

Corollary. In case of linear cost functions and linear inverse functions of demand optimal modified, two-part trade policy maximizing the domestic welfare dominates on other trade policy instruments.

6. Choice of trade policy instruments.

At the first level governments announce a trade policy, and on the second,- home and foreign producers behave as Cournot competitors.

Let's consider three step game. At the first level governments announce a trade policy instrument, on the second governments set a concrete value of parameters of this trade policy, and on the third,- home and foreign producers behave as Cournot competitors.

On a first step of government can choose the following policies:

1) $\tilde{z}_0=(0,\infty,0,0,\infty,0)$ - free trade; 2) $\tilde{z}_1=(0,\infty,t,0,\infty,0)$ - simple tariff on exterior firm;

3) $\tilde{z}_2=(0,\infty,0,0,\infty,t_1)$ - simple tariff on internal firm; 4) $\tilde{z}_3=(0,\infty,t,0,\infty,t_1)$ - simple tariff on exterior and internal firms; 5) $\tilde{z}_4=(e,\overline{q},t,0,\infty,0)$ - two-part trade policy; 6) $\tilde{z}_5=(e,\overline{q},t,0,\infty,t_1)$ - modified two-part trade policy. Interaction of trade policy instruments 1) -6) we shall consider on examples.

Example 6.1. Let's assume are known inverse demand function in the home and foreign market and cost functions of firms:

$p(Q)=1-Q$, $Q=q_1+q_2$; $p^*(Q^*)=1-Q^*$, $Q^*=q_1^*+q_2^*$; $C_1(q)=0.1\cdot q$; $C_1^*(q)=0.1\cdot q$.

Thus the governments maximize domestic welfare.

Optimal trade policies for every possible combinations of choices of governments we reduce in the table 6.1

The analysis of the table 6.1 shows that optimal trade policy will be two-part trade policy or modified two-part trade policy. For a case $(\tilde{z}_5, \tilde{z}_5^*)$ ***there is an infinitely many of optimal trade policies and is defined by a conditions*** $t + t_1 = -0.9$***;*** $t^* + t_1^* = -0.9$ ***(for this case in the table 6.1 one of possible variants is reflected only). Two-part trade policy and modified two-part trade policy are equivalent concerning a maxima of domestic welfare. However at the modified two-part trade policy the payment for the license is much lower, and the profit is distinct from zero.***

Proposition 6. In case of nonlinear cost functions and linear inverse functions of demand modified two-part trade policy maximizing the domestic welfare dominates on other trade policy instruments.

The proof of the given statement is not present. Let's illustrate the given proposition by an example.

Example 6.2. Let's assume are known inverse demand function in the home and foreign market and cost functions of firms:

$$p(Q) = 1 - Q\,,\ Q = q_1 + q_2\,;\ \ p^*(Q^*) = 1 - Q^*,\ Q^* = q_1^* + q_2^*;\ \ C_1(q) = 0.5 \cdot q^2\,;\ \ C_1^*(q) = 0.5 \cdot q^2\,.$$

Thus the governments maximize domestic welfare.

Optimal trade policies for every possible combinations of choices of governments we reduce in the table6.2.

The analysis of the table 6.1 shows that optimal trade policy will be modified two-part trade policy. At the modified two-part trade policy the profit of firms payment is more and the market price of the goods is lower than for other instruments.

7. Conclusion

What is the optimum mix of trade policy instruments? In the report the possibility of a simultaneous use by the government of quotas (and corresponding License fees) and tariffs is analyzed. The use of quotas and tariffs as complements rather than substitutes allows to carry out a trade policy which dominates – from the efficiency point of view - over a policy based on others trade policy instruments. The qualitative outcomes of the analysis depend on the type of government (whether it maximizes its revenue or public welfare), market structure and the cost structure of firms operating in the market.

The report analyses the problems of realization of the state two-part trade policy in case of two countries, two markets, many producers in conditions of imperfect competition.

We have shown that the two-part trade policy dominates the simple quota and simple tariff, and the last two mentioned are the special cases of it. Thus the effectiveness of the two-part trade policy depends on the number of competing firms on the market, magnitude of a heterogeneity of functions of costs, degrees of convexity of functions of costs and thus to what firms the given policy will be applied and the governments of which countries will execute the policy.

In the beginning we have shown, that for the third-market model the optimal two-part trade policy is a subsidy. For this case the analytical results (Lemma 4.1 and Theorem 4.1) are obtained. In this part we investigated substantial sense of properties of cost functions and inverse demand functions which define existence of optimal trade policy.

In model of the reciprocal markets with homogeneous structure of cost functions the conditions of existence of optimal trade policy, as for a case of a maximization of the governmental income (Theorem 4.2), and for a case of a maximization of the domestic welfare (Theorem 4.3) are found. All these results are illustrated with a set of examples. For a homogeneous case we have established that

1) A subsidy appear (both for G-government and for W-government) more often and under other conditions, than in Fuerst and Kim models and it depends on a difference among foreign and home firms and from a cost value.

2) ***For G-government, when all firms have high costs, independently of a number of firms in the market the optimal policy is a subsidy;***

3) At a internal government (in case of G) the optimal policy appear through a positive tariff, when all firms have low costs, and a number of external firms is more than internal firms on 2 or more. Thus the optimal policy of a external government will be always a subsidy;

4) At a internal government (in case of W) the optimal policy will be always a subsidy, if there is more internal firms in than the market, than external firms. Thus the optimal policy of a external government will be a subsidy at low costs and a positive tariff at high costs. In case of equality of a number of firms in the market the optimal policy is always a subsidy.

In model of the reciprocal markets with inhomogeneous structure of cost functions on the basis of numerical modeling in a package Maple we have established that :

1) For G-government at any degree of a heterogeneity the internal government will establish the high payment of an entrance for external firms before, than its the policy will be replaced from a subsidy with a positive tariff, i.e. the optimal policy will be always a subsidy.

2) For W-government the situation is similar, except for a case when a number of external and internal firms is equal to one, and the costs of internal firms exceed costs of external firms, then the optimal policy of internal government is a positive tariff.

We have analysed also negative aspect of the two-part trade policy, which consists of the fact that in conditions of transitional economy the given policy has a negative consequence for the internal producers. In the first place, falling the decrease of internal production when only one government using a two-part tariff. It's worth reminding that a government which maximizes incomes, encourages import firms to produce Stackelberg output level. And in case of maximization of domestic welfare we have competitive outcome, but with constant sharp reduction of internal production, and consequently by a greater dependence of a country on export.

The modified two-part trade policy includes applying the two-part trade policy to the internal producers as well, that in case of oligopoly it has reduced in the subsidy (as well as for the foreign producers). In case of maximization of government incomes the modified two-part trade policy is not the subsidy at any conditions, though the protection of the domestic producer is possible in this case. In case of a maximization of domestic welfare the modified policy is the subsidy.

The results of comparison of trade policy instruments show that the modified two-part policy is dominating.

The two-part trade policy is considered for Bertrand competition.

There are two countries, one home, other foreign. There is one home and one foreign firm. Let p_i - price of the goods firm i on the home market; Let p_i^* - price of the goods firm i in the foreign market; The market demand function is $q = D(p)$ and $q^* = D^*(p^*)$ in the home and foreign market. Each firm incurs a cost c_i per unit of production ($c_1 < c_2$). Therefore, the profit of firm i is

$$\pi_1 = (p_1^* - c_1)\cdot D_1^*(p_1^*, p_2^*) - t^* \cdot D_1^*(p_1^*, p_2^*) - e^* + (p_1 - c_1)\cdot D_1(p_1, p_2),$$

$$\pi_2 = (p_2 - c_2)\cdot D_2(p_1, p_2) - t \cdot D_2(p_1, p_2) - e + (p_2^* - c_2)\cdot D_2^*(p_1^*, p_2^*),$$

where the demand for the output of firm i, denoted D_i, is given by

$$D_i = \begin{cases} D(p_i) & \text{if } p_i < p_j \\ \frac{1}{2}D(p_i) & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j \end{cases}; \qquad D_i^* = \begin{cases} D_i^*(p_i) & \text{if } p_i^* < p_j^* \\ \frac{1}{2}D_i^*(p_i) & \text{if } p_i^* = p_j^* \\ 0 & \text{if } p_i^* > p_j^* \end{cases};$$

e, e* - payment for the license to home and foreign government accordingly; t, t*- the tariff per unit of production imposed on home and foreign firms accordingly.

The payoff functions of governments are defined similarly of formulas (4.24) and (4.25).

Because of market segmentation and because of the constancy of marginal cost, we can proceed by examining just one national market. Then

$\pi_1 = (p_1^* - c_1) \cdot D_1^*(p_1^*, p_2^*) - t^* \cdot D_1^*(p_1^*, p_2^*) - e^*$,

$\pi_2 = (p_2^* - c_2) \cdot D_2^*(p_1^*, p_2^*)$.

Lemma 5. In given Bertrand model optimal two-part policy is the positive tariff

$t^* = \begin{cases} (c_2 - c_1), & if \quad c_2 \le p^m(c_1) \\ (p^m(c_1) - c_1), & if \quad c_2 > p^m(c_1) \end{cases}$, where $p^m(c_1)$ maximizes $(p - c_1) \cdot D(p)$.

The optimal policy in given Bertrand model differs from optimal policy in Cournot model. So in Cournot model for a case of two firms optimal two-part policy always was subsidy, which with growth of number of firms passed in the positive tariff. In Bertrand model for a case of **two firms** as optimal policy we have the positive tariff (not subsidy).

Have two-part trade policy actually been used in the international trade? Have it been used in the market structures analyzed in the project? The answer is yes.

There is a defined number of examples of use of such policies:

1) Two-part tariffs (TPT) (It is a case of two-part trade policy $z = (e, \infty, t)$)

Two-part tariffs are pricing schemes that involve a fixed fee which must be paid to consume any amount of good, and then a variable fee based on usage. Two-part tariffs are commonly used in practice. Table TPT gives a few examples:

Table TPT.

	Fixed premium	Charge varying according to
Telephone, gas, electricity	Rental	Number of units
Polaroid camera	Camera purchase	Amount of film
Amusement park	Входной билет	Number of rides
Taxi	Initial meter reading	Distance

In international trade the two-part tariff is used by governments to such firms:

the suppliers of the electric power, services of cellular connection, Internet of services, services of transport.

2) Tariff-rate quota (TCR) (It is a case of two-part trade policy $z = (0, \overline{q}, t)$)

A tariff-rate quota is a quota for a volume of imports at a lower tariff. After the quota is reached, a higher tariff is applied on additional imports. Suppose a country replaces its quota of 10,000 tons with a TRQ of 10,000 tons. The TRQ appears to differ from the "absolute" quota. The distinction is that under an absolute quota it is legally impossible to import more than 10,000 tons, whereas under a TRQ, imports can exceed 10,000 tons but a higher, over-quota tariff is applied on the excess. In principle, a TRQ provides more market access to imports than a quota. In practice, however, many over-quota tariffs are prohibitively high and effectively exclude imports in excess of the quota. It is possible to design a TRQ so that it reproduces the trade-volume limit of the quota it replaces.

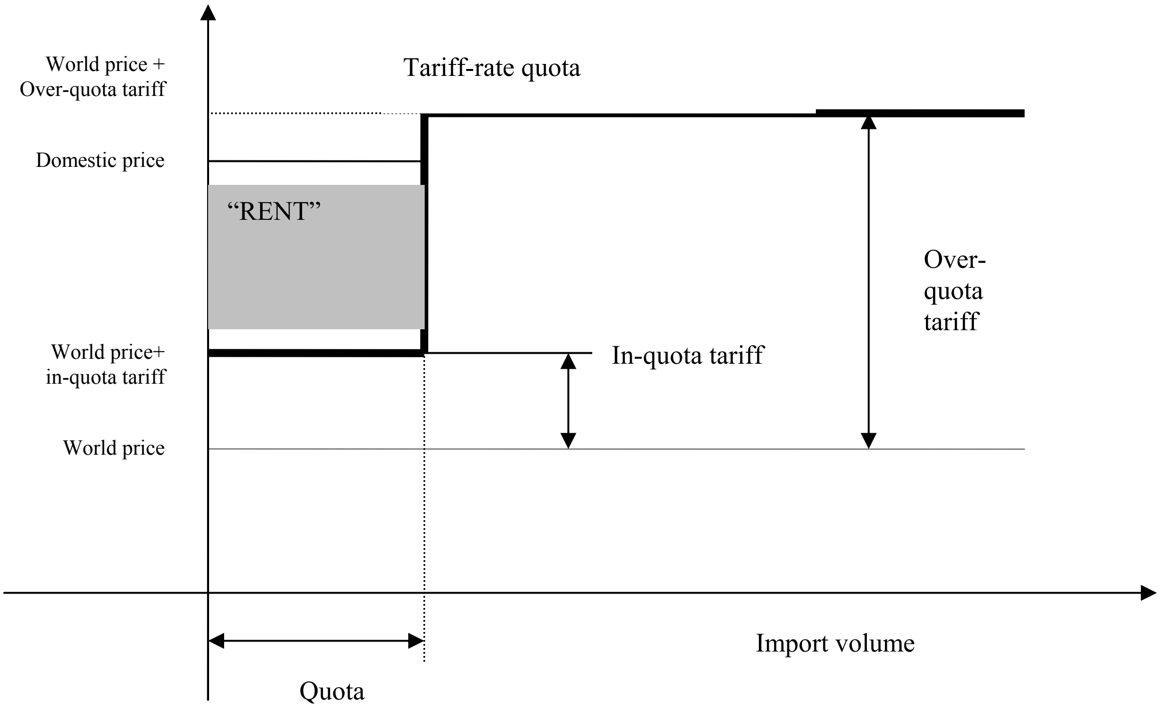
Tariff-rate quotas (TRQ's) are two-level tariffs. TRQ's were adopted during the Uruguay Round as a method for providing greater access to markets with high tariffs. A limited volume of imports is allowed at the lower tariff, and all subsequent imports are charged the higher tariff. If the demand for imports at the low tariff is greater than the volume allowed by the TRQ, then imports must be rationed.

A TRQ has three components:

- a quota that defines the maximum volume of imports charged the in-quota tariff,
- an in-quota tariff, and
- an over-quota tariff.

The two-level tariff results in a stepped import supply function. Imports within the quota are charged the lower tariff; over-quota imports are charged the higher tariff. This results in a vertical step when the quota volume is filled. The figure illustrates a case in which domestic demand is sufficient to import the full quota volume at the in-quota tariff, but the over-quota tariff is prohibitive. That is, the domestic price is below the price of imports with the over-quota tariff, thus there is no incentive to import beyond the quota. Were domestic demand to increase, it might become profitable to import at the over-quota tariff. This opportunity would not be possible with a standard, absolute quota.

TRQ administration involves distributing the rights to import at the in-quota tariff. Whoever obtains such rights can make a risk-free profit of the difference between the domestic price, and the world price inclusive of the in-quota tariff. The area labeled 'RENT' in the figure represents the value of these profitable opportunities. Rents indicate that the demand to trade within the quota is greater than the supply of quota; thus the necessity to ration or administer the TRQ.



Which countries have TRQ's? Of the 137 WTO members, 37 countries permanently use TRQ's. The countries with the greatest number of TRQ's are concentrated among relatively wealthy economies with historically protectionist agricultural policies. In addition, several Central and Eastern European countries have adopted TRQ's to ease the transition of their agricultural sectors into a market-oriented economy. Tariff quota administration is fundamentally a rationing problem. The issue for TRQ administration is to determine whether some ways of rationing are better than others.

The following example is considered: "Russia Threatens Tariff and Quota Hikes" that can illustrate the theoretical part of study:

Meat and poultry exporters to Russia face heavy tariffs next year if plans by the Russian Agriculture Ministry come in to force. The ministry has said it intends to raise import tariffs as a protective measure and impose import quotas on beef, pork and poultry. The ministry said the measures are aimed to protect the domestic industry from dumping of cheap imports. The tariffs come as part of the Russian retaliation to grain quotas that have been imposed by the EU and which are expected to effect exports from Russia.

It is believed that the new tariff on pork will be set at 80 per cent compared to the present 15 per cent. On beef the tariff will rise from 15 per cent to 25 per cent and on poultry from 25 per cent to 35 per cent. The quotas that are being proposed would limit imports of pork to Russia to 340,000 tones, beef 420,000 tones and poultry 750,000 tones.

Last year Russia imported 348,000 tones of pork of which 315,000 tones came from the EU, mainly from Denmark with some from Germany, France and the Netherlands, and 27,600 tones coming from the US. Russia's pork imports amounted to a quarter of the country's own output Russia also imported 475,000 tones of beef and 1.39 million tones of poultry meat.

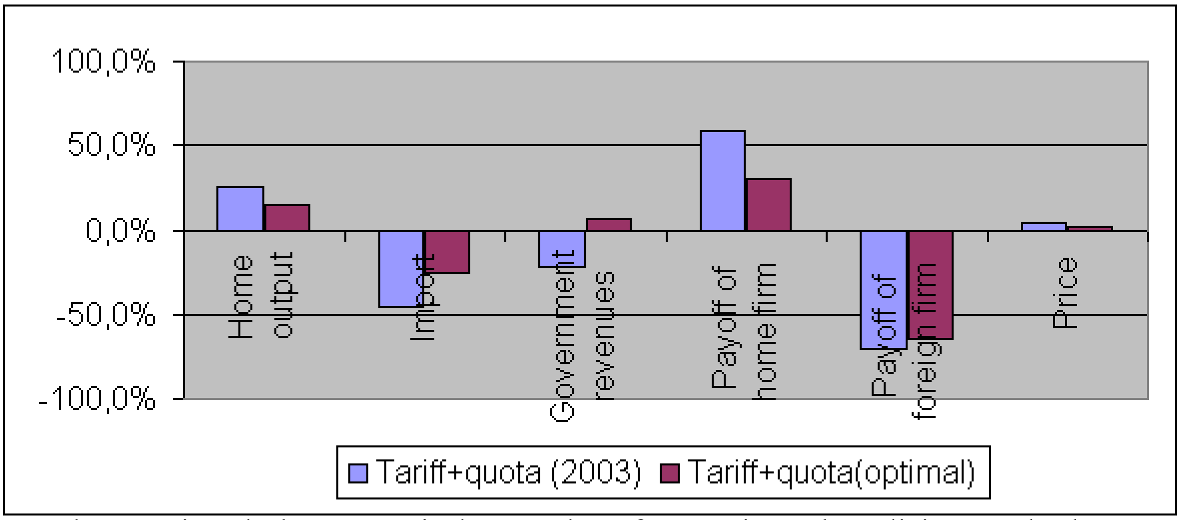
Below in the table 7.1 the comparative analysis on the poultry meat for 2003 and optimum two-part policy with results of the simple tariff of 2001 is given.

Tab 7.1

Poultry meat

	Simple tariff (2001)	Tariff+quota (2003)		Tariff+quota (optimal)	
	t=25%	t=35%,q=0,75		t=35%,q=1,030	
Home output	1,240	1,560	25,8%	1,420	14,5%
Import	1,390	0,750	-46,0%	1,030	-25,9%
Government revenues	0,264	0,207	-21,6%	0,281	6,4%
Payoff of home firm	0,140	0,222	58,6%	0,183	30,7%
Payoff of foreign	0,176	0,052	-70,5%	0,063	-64,2%

firm					
Price	0,760	0,789	3,8%	0,777	2,2%



The mentioned above practical examples of strategic trade policies speak about actual of the theoretical results, obtained in the project.

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9. Appendices. The proofs of the theorems.

Proof of Lemma 4.1.:

The solution to the first order conditions (4.5), (4.6) will yield q_1 and v_1 as functions of tariffs t and t*. The comparative static effects $\frac{\partial q_1}{\partial t}, \frac{\partial q_1}{\partial t^*}, \frac{\partial v_1}{\partial t}, \frac{\partial v_1}{\partial t^*}$ can be obtained by totally differentiating first order conditions (4.7) with respect to q_1, v_1, t, t* as follows.

$$\begin{cases} \frac{\partial^2 \pi_1}{\partial q_1^2} \cdot \frac{\partial q_1}{\partial t} + \frac{\partial^2 \pi_1}{\partial q_1 \partial v_1} \cdot \frac{\partial v_1}{\partial t} = 1 \\ \frac{\partial^2 \pi_1^*}{\partial v_1 \partial q_1} \cdot \frac{\partial q_1}{\partial t} + \frac{\partial \pi_1^*}{\partial v_1 \partial v_1} \cdot \frac{\partial v_1}{\partial t} = 0 \end{cases} \quad (4.10) \text{ and } \quad \begin{cases} \frac{\partial^2 \pi_1}{\partial q_1^2} \cdot \frac{\partial q_1}{\partial t^*} + \frac{\partial^2 \pi_1}{\partial q_1 \partial v_1} \cdot \frac{\partial v_1}{\partial t^*} = 0 \\ \frac{\partial^2 \pi_1^*}{\partial v_1 \partial q_1} \cdot \frac{\partial q_1}{\partial t^*} + \frac{\partial \pi_1^*}{\partial v_1 \partial v_1} \cdot \frac{\partial v_1}{\partial t^*} = 1 \end{cases} \quad (4.11)$$

These equations can be solved using Cramer's rule:

$$\frac{\partial q_1}{\partial t} = \frac{1}{D} \cdot \frac{\partial^2 \pi_1^*}{\partial v_1 \partial v_1}; \ \frac{\partial v_1}{\partial t} = -\frac{1}{D} \cdot \frac{\partial^2 \pi_1^*}{\partial v_1 \partial q_1}; \ \frac{\partial q_1}{\partial t} = -\frac{1}{D} \cdot \frac{\partial^2 \pi_1}{\partial q_1 \partial v_1}; \ \frac{\partial q_1}{\partial t} = \frac{1}{D} \cdot \frac{\partial^2 \pi_1}{\partial q_1 \partial q_1},$$

where $D = \frac{\partial^2 \pi_1}{\partial q_1 \partial q_1} \cdot \frac{\partial^2 \pi_1^*}{\partial v_1 \partial v_1} - \frac{\partial^2 \pi_1}{\partial q_1 \partial v_1} \cdot \frac{\partial \pi_1^*}{\partial v_1 \partial q_1}$ is the determinant of the left-hand matrix in (4.10) and (4.11).

From (4.8) and (4.9), D>0; $\frac{\partial q_1}{\partial t} < 0, \frac{\partial q_1}{\partial t^*} > 0, \frac{\partial v_1}{\partial t} > 0, \frac{\partial v_1}{\partial t^*} < 0$. (4.12)

<u>Proof of Corollary 4.1.:</u>

Let $\tilde{q}_1(v_1)$ and $\tilde{v}_1(q_1)$ be domestic firm's best response and foreign firm's best response.

By the first-order condition $\frac{\partial \tilde{v}_1}{\partial q_1} = -\frac{p'' \cdot v_1 + p'}{2p' + p'' \cdot v_1 - c_1^{*}{}''} = \alpha$. Then, using conditions 1)-3) of Lemma 4.1., we get

$-1 < \alpha < 0$. Next, we have that $\frac{\partial v_1}{\partial t} = \frac{\partial \tilde{v}_1}{\partial q_1} \cdot \frac{\partial q_1}{\partial t} = \alpha \cdot \frac{\partial q_1}{\partial t}$. It is similarly proved that $\frac{\partial q_1}{\partial t^*} = \beta \frac{\partial v_1}{\partial t^*}$, where $\beta \in (-1,0)$.

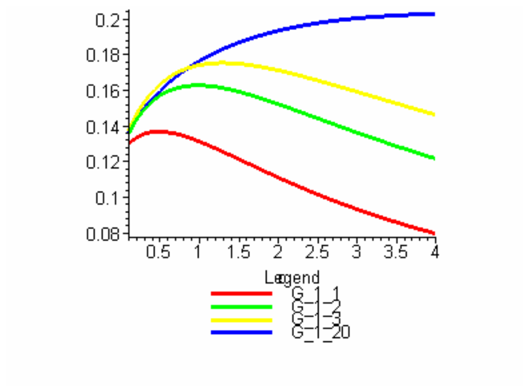
Tab.4.1

	Home government		
Foreign government		$z = (0,\infty,0)$ free trade	$z = (e,\bar{v},t)$ two-part trade

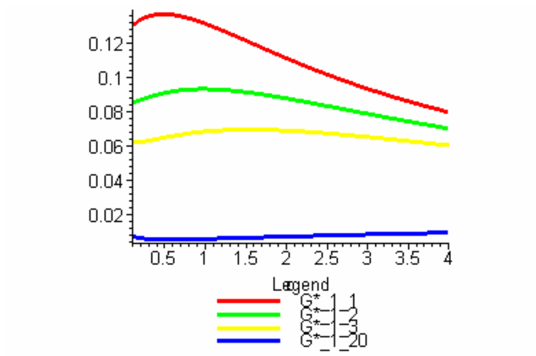
	$z^* = (0,\infty,0)$
	free trade
		<u>G=0, π=(a-2c*+c)^2/9b;</u>
		<u>G*=0, π*=(a-2c+c*)^2/9b;</u>
		q=(a-2c+c*)/3b;q*=(a-2c*+c)/3b;
		t=0;t*=0;e=0;e*=0;
			<u>G=(a-2c+c*)^2/8b, π=0;</u>
			<u>G*=0, π*=(a-3c*+2c)^2/16b;</u>
			q=(a-2c+c*)/2b;q*=(a-3c*+2c)/4b;
			t=-(a-2c+c*)/4,t*=0;e=(a-2c+c*)^2/4b;e*=0;

	$z^* = (e^*, \overline{q}, t^*)$ two-part	G=0, π=(a-3c+2c*)^2/16b;
		G*=(a-2c*+c)^2/8b, π*=0;
		q=(a-3c+2c*)/4b; q*=(a-2c*+c)/2b;
		t=0; t*=-(a-2c*+c)/4b;
		e=0; e*=(a-2c*+c)^2/4b;
			G=2(a-3c+2c*)^2/25b, π=0;
			G*=2(a-3c*+2c)^2/25b, π*=0;
			q=2(a-3c*+2c)/5b, q*=2(a-3c+2c*)/5b;
			t=-(a-3c+2c*)/5,t*=-(a-3c*+2c)/5;
			e=4(a-3c*+2c)^2/25b;e*=4(a-3c+2c*)^2/25b;

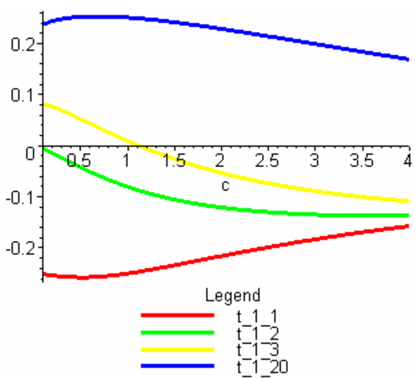
Example 4.3. (homogeneous case) Two markets two governments ((in case of G), and three cases: 1) two firms (N=1, N * =1); 2) three firms (N=1, N * =2); 3) four firms (N=1, N *=3); 4) many firms (N=1, N *=20). Inverse functions of demand in the home and foreign market: p(Q)=1-Q; p*(Q*)=1-Q*; the costs functions of home and foreign firms: C(q) =cq^2/2; C*(q) =cq^2/2. Outcomes of optimum policy for various values of parameter c=0.. 4 were simulated in Maple 7 and are illustrated on the graphs:



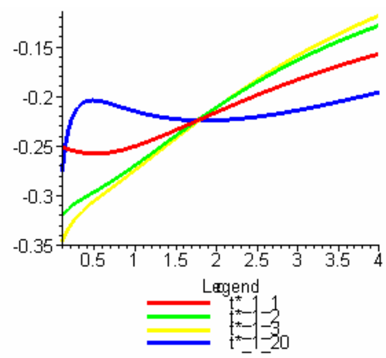
income of home government



income of foreign government



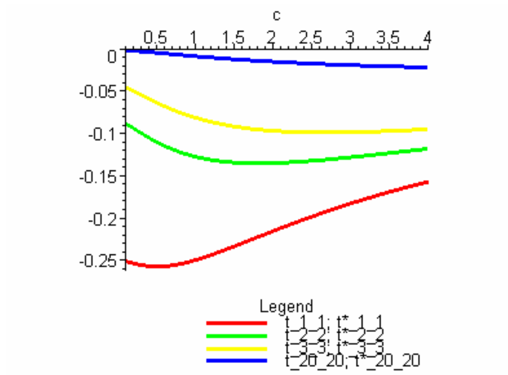
optimal two-part tariff of home government



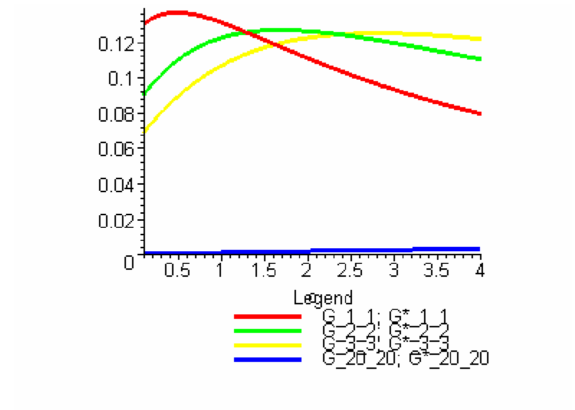
optimal two-part tariff of foreign government

Example 4.4. Two markets two governments ((in case of G), and three cases: 1) two firms (N=1, N * =1); 2) three firms (N=2, N * =2); 3) four firms (N=3, N *=3); 4) many firms (N=20, N *=20). Inverse functions of demand in the home and foreign market: p(Q)=1-Q; p*(Q*)=1-Q*; the costs functions of home and foreign firms: C(q) =$cq^2/2$; C*(q) =$cq^2/2$.

Outcomes of optimum policy for various values of parameter c=0.. 4 were simulated in Maple 7 and are illustrated on the graphs:



optimal two-part tariff of home government & optimal two-part tariff of foreign government



income of home government & income of foreign government

Example 4.5. Two markets, two governments, two firms. All parameters are similar to an Example 4.2 except for elimination c=0,5 (for simplification of the comparative analysis). The optimum policies for different combinations of choices of governments are given in Tab. 4.3.

		Home government		Tab. 4.3.
		$z=(0,\infty,0)$ free trade	$z=(0,\infty,t)$ simple tariff	$z=(e,\bar{v},t)$ two-part trade

Foreign government	$z^*=(0,\infty,0)$
	free trade
		G=0; π=0,1875;
		G*=0; π*=0,1875;
		q=0,25; q*=0,25;
		v=0,25; v*=0,25;
		t=0; e=0;
		t*=0; e*=0;
			G=0,03; π=0,1389;
			G*=0; π*=0,2139;
			q=0,2850; q*=0,1250;
			v=0,2250; v*=0,3050;
			t=0; e=0;
			t*=0,2400; e*=0;
				G=0,1264; π=0,1665;
				G*=0; π*=0,0548;
				q=0,1862;q*=0,2790;
				v=0,3951;v*=0,2094;
				t=-0,2785;e=0,2364;
				t*=0;e*=0;

	$z^* = (0, \infty, t^*)$ simple tariff	G=0; π=0,2139; G*=0,03; π*=0,1389; q=0,2250;q*=0,3050; v=0,2850;v*=0,1250; t=0,2400;e=0; t*=0;e*=0;	G=0,0248; π=0,1726; G*=0,0248;π*=0,1726; q=0,3318;q*=0,1136; v=0,3318;v*=0,1136; t=0,2182;e=0; t*=0,2182;e*=0;	G=0,1160; π=0,4011; G*=0,0380;π*=0,5913; q=0,2205;q*=0,1407; v=0,3784;v*=0,2681; t=-0,3005;e=0,2297 t*=0,2701;e*=0;
	$z^* = (e^*, \overline{q}, t^*)$ two-part trade	G=0; π=0,0548 G*=0,1264;π*=0,1665; q=0,3951;q*=0,2094; v=0,1862;v*=0,2790; t=0;e=0; t*=-0,2785;e*=0,2364;	G=0,0380; π=0,5913; G*=0,1160; π*=0,4011; q=0,3784;q*=0,2681; v=0,2205;v*=0,1407; t=0,2701;e=0,2297; t*=-0,3005;e*=0;	G=0,1368; π=0,0294; G*=0,1368; π*=0,0294; q=0,1534;q*=0,4110; v=0,4110;v*=0,1534; t=-0,2575;e=0,2426; t*=-0,2575;e*=0,2426;

The considered example is also new, as it is introduced for the first time for the two-part trade policy in case of two countries and two markets, though is compared to known results for a case of free trade and simple tariff. The references are not specified as the given outcomes were obtained by the author, as a special case of the two-part trade policy. From the given example it is visible, that the optimum two-part trade policy is the subsidy at the positive payment for the license.

From the given example it is visible, that an optimum two-part trade policy is the subsidy at the positive payment for the license.

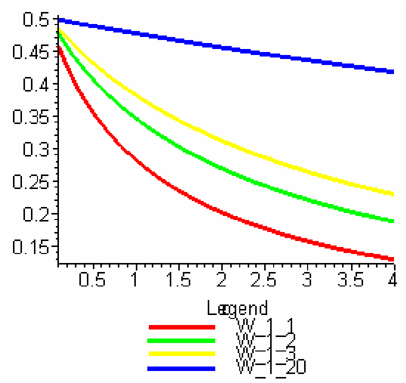
If the two-part trade policy is applied only by one government, the optimum subsidy of exterior firm moves it on the leading Stakelberg output level, and the interior firm is moved to the level of the follower. Thus the income of government at the two-part policy is the largest.

If the two-part policy is applied by both governments (at a maximization Gk), their optimum policy still will be the subsidy at the positive tariff. However in case of maximization of welfare (Gk $+\pi_k$) the game at a level of governments looks like a prisoners` dilemma made, as both producing countries are worse off at the strategic subsidy equilibrium than they would be under free trade, but each has a unilateral incentive to intervene. Thus this deviation goes not on the simple tariff, and on the two-part trade policy, which obviously dominates above the simple tariff.

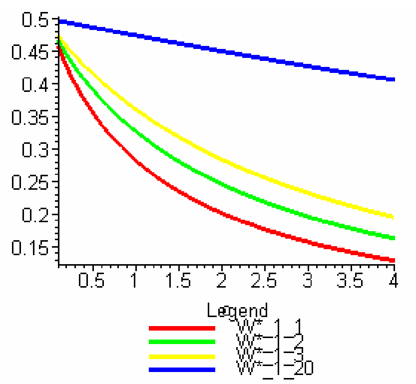
Let's consider for an example 4.5, whether the optimum two-part trade policy of the Pareto-optimal with respect to the policies of each government (G+G *), i.e. the case when between governments is present collusion. In this case we have the following first-best outcome: t=t*=-0.3385<0, which rather differs from the individually two-part tariffs t=t*=-0.2575<0 (G=G*=0.1368) . In case when between governments is present collusion G=G*=0.1385>0.1368, q=0.1231,q*=0.4615; v=0.1231,v*=0.4615;e= e*=0.2947. Here, the game also has the prisoners' dilemma flavor in the sense that collusion between the two governments would have resulted in greater revenue for each (assuming equal division of total revenue) that individually rational policies.

<u>Example 4.6.</u> Two markets two governments ((in case of W), and three cases: 1) two firms (N=1, N * =1); 2) three firms (N=1, N * =2); 3) four firms (N=1, N *=3); 4) many firms (N=1, N *=20).. Inverse functions of demand in the home and foreign market: p(Q)=1-Q; p*(Q*)=1-Q*; the costs functions of home and foreign firms: C(q) =$cq^2/2$; C*(q) =$cq^2/2$.

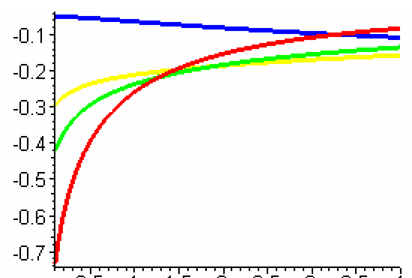
Outcomes of optimum policy for various values of parameter c=0.. 4 were simulated in Maple 7 and are illustrated on the graphs:

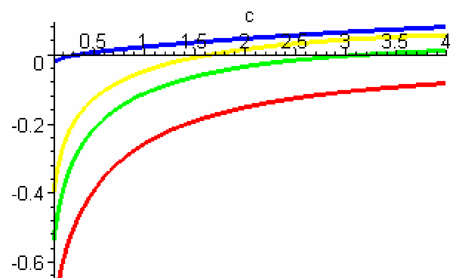


domestic welfare of a home country



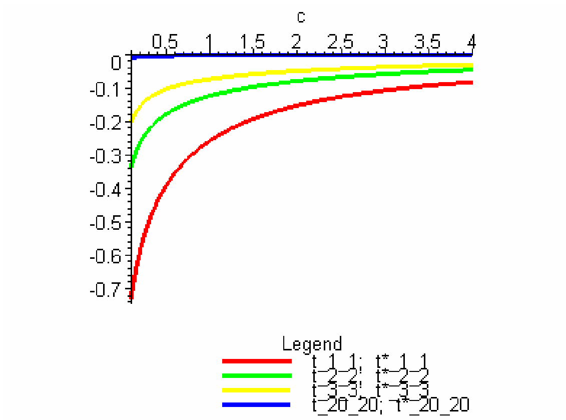
domestic welfare of a foreign country



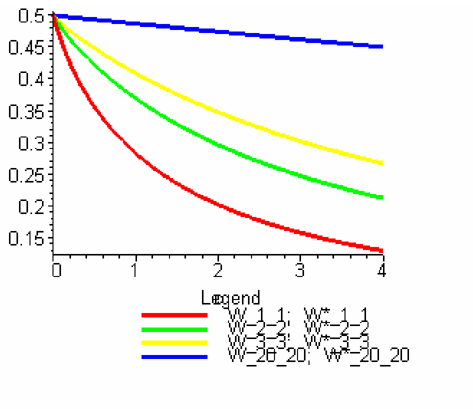


Example 4.7. Two markets two governments ((in case of W), and three cases: 1) two firms (N=1, N * =1); 2) three firms (N=2, N * =2); 3) four firms (N=3, N *=3) ; 4) many firms (N=20, N *=20).. Inverse functions of demand in the home and foreign market: p(Q)=1-Q; p*(Q*)=1-Q*; the costs functions of home and foreign firms: $C(q) = cq^2/2$; $C^*(q) = cq^2/2$.

Outcomes of optimum policy for various values of parameter c=0.. 4 were simulated in Maple 7 and are illustrated on the graphs:



optimal two-part tariff of home government & optimal two-part tariff of foreign government

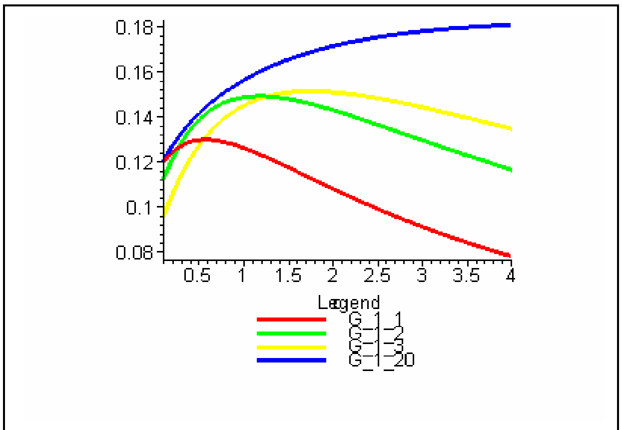


domestic welfare of a home country & domestic welfare of a foreign country

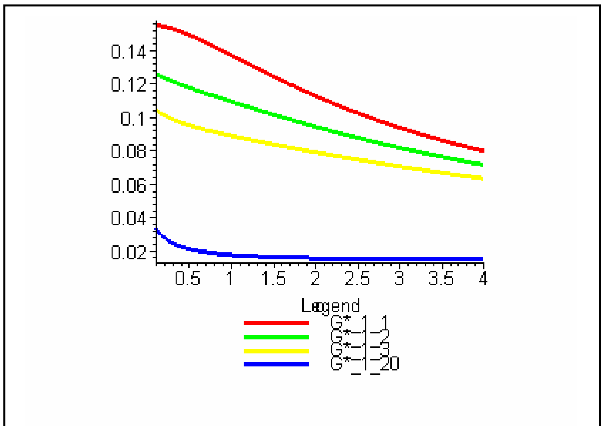
Example 4.8. (inhomogeneous case) Two markets, two governments (in case of G), and three cases::1) two firms (N=1, N*=1);) three firms (N=1, N=2); 3) four firms (N=1, N*=3); 4) many firms (N=1, N*=20). Inverse functions of demand in the home and foreign market: p(Q)=1-Q; p*(Q*) = 1-Q*; the costs functions of home and foreign firms: $C_i(q)= c_i q^2/2$, i=1..N; $C_j^*(q) = c_j^* q^2/2$, j=1..N* ,

$c_1 = c \le c_1^* = c + 0.1 < c_2^* = c + 0.2 < c_3^* = c + 0.3 < c_{20}^* + 2$. Here increase of parameter c reduces in a decrease of a heterogeneity of costs functions.

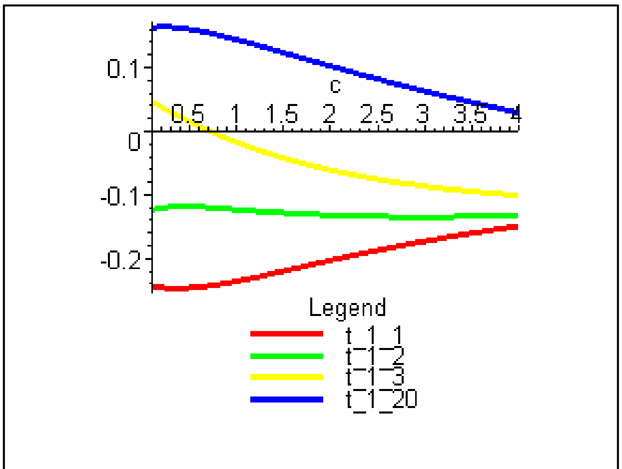
Outcomes of optimum policy for various values of parameter c=0.. 4 were simulated in Maple 7 and are illustrated on the graphs:



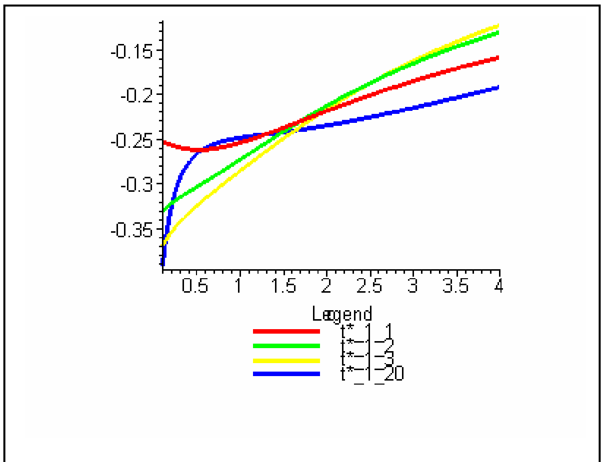
income of home government



income of foreign government



optimal two-part tariff of home government

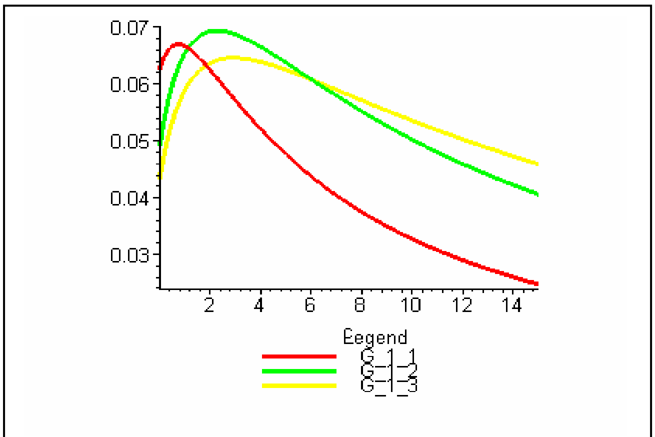


optimal two-part tarifff of foreign government

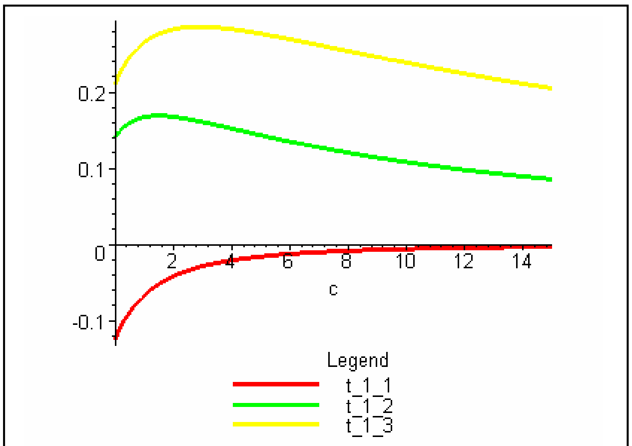
As in an inhomogeneous case in work Fuerst and Kim was considered only one numerical example, for the purposes of comparison with our model we shall conduct more account of models Fuerst and Kim for the greater number of cases (for different N, N *, C_i and C_j^*).

Example 4.9. (inhomogeneous case, Fuerst and Kim) One market, one government (in case of G), and three cases: 1) two firms (N=1, N*=1); 2) three firms (N=1, N=2); 3) four firms (N=1, N*=3). Inverse functions of demand in the home and foreign market: p(Q)=1-Q; p*(Q*) = 1-Q*; the costs functions of home and foreign firms: $C_i(q)= c_i\ q^2/2$, i=1..N; $C_j^*(q) = c_j^* q^2/2$, j=1..N*, $c_1 = c \le c_1^* = c+1 < c_2^* = c+3 < c_3^* = c+9$. Here increase of parameter c reduces in a decrease of a heterogeneity of costs functions.

Outcomes of optimum policy for various values of parameter c=0,1.. 15 were simulated in Maple 7 and are illustrated on the graphs:



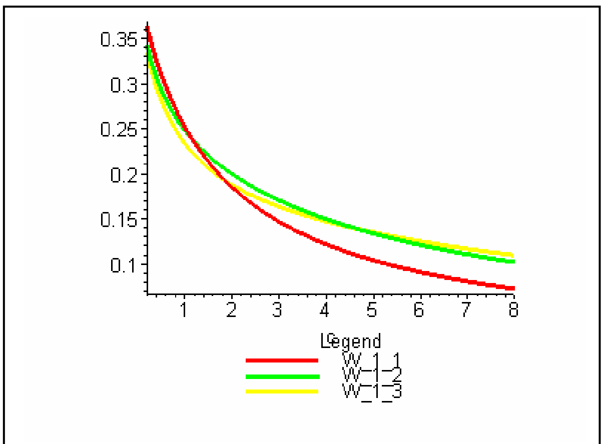
income of home government

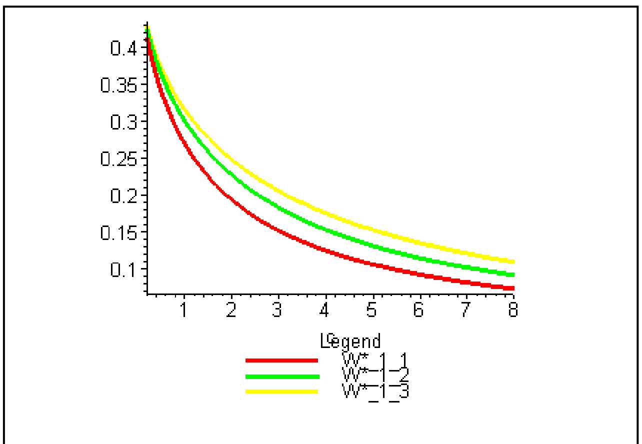


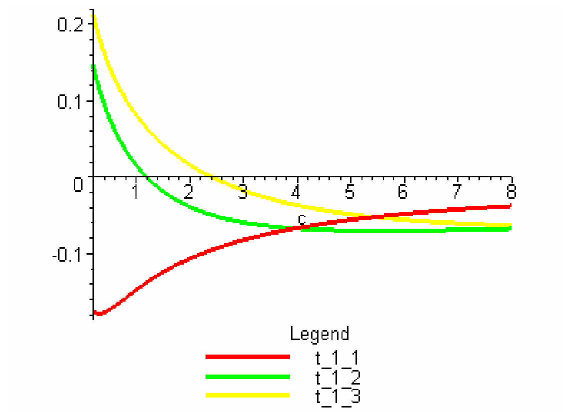
optimal two-part tariff of home government

<u>Example 4.10.</u> (inhomogeneous case**)** Two markets, two governments (in case of W), and three cases: 1) two firms (N=1, N*=1); 2) three firms (N=1, N=2); 3) four firms (N=1, N*=3). Inverse functions of demand in the home and foreign market: p(Q)=1-Q; p*(Q*) = 1-Q*; the costs functions of home and foreign firms: $C_i(q)= c_i\ q^2/2$, i=1..N; $C_j^*(q) = c_j^* q^2/2$, j=1..N*, $c_1 = c \le c_1^* = c+0{,}5 < c_2^* = c+1{,}5 < c_3^* = c+3$. Here increase of parameter c reduces in a decrease of a heterogeneity of costs functions.

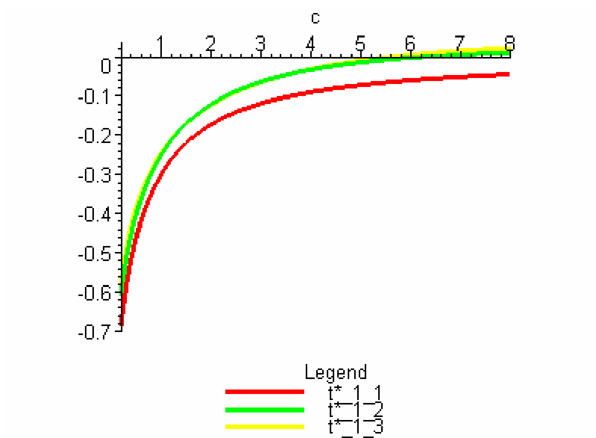
Outcomes of optimum policy for various values of parameter c=0,2.. 8 were simulated in Maple 7 and are illustrated on the graphs:







optimal two-part tariff of home government

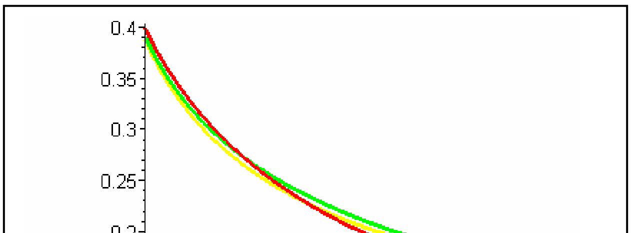


optimal two-part tariff of foreign government

As in an inhomogeneous case in work Fuerst and Kim was considered only one numerical example, for the purposes of comparison with our model we shall conduct more account of models Fuerst and Kim for the greater number of cases (for different N, N *, C_i and C_j^*).

<u>Example 4.11.</u> (inhomogeneous case, Fuerst and Kim) One market, one government (in case of W), and three cases: 1) two firms (N=1, N*=1); 2) three firms (N=1, N=2); 3) four firms (N=1, N*=3). Inverse functions of demand in the home and foreign market: p(Q)=1-Q; p*(Q*) = 1-Q*; the costs functions of home and foreign firms: $C_i(q) = c_i\, q^2/2$, i=1..N; $C_j^*(q) = c_j^* q^2/2$, j=1..N*, $c_1 = c \le c_1^* = c+1 < c_2^* = c+3 < c_3^* = c+9$. Here increase of parameter c reduces in a decrease of a heterogeneity of costs functions.

Outcomes of optimum policy for various values of parameter c=0,1.. 4 were simulated in Maple 7 and are illustrated on the graphs:



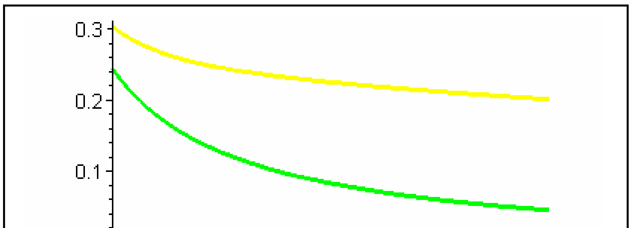


Table 6.1

	$\tilde{z}_0$	$\tilde{z}_1$	$\tilde{z}_2$	$\tilde{z}_3$	$\tilde{z}_4$	$\tilde{z}_5$
z_0	**W=0.36,W*=0.36** t = -----, t_1 = ---- t*= -----, t^*_1= ---- q_1=0.30,q1*=0.30 q_2=0.30,q2*=0.30 e = ---,e* = --- π_1=0.18,π_2=0.18 p=0.40,p*=0.40	**W=0.36,W*=0.34** t = -----, t_1 = ---- t*= -----, t^*_1=0.23 q_1=0.38,q1*=0.30 q_2=0.15,q2*=0.30 e = ---,e* = ---- π_1=0.23, π_2=0.11 p =0.40,p*=0.48	**W=0.36,W*=0.34** t = ----, t_1 = ---- t*= 0.23, t^*_1=---- q_1=0.15,q1*=0.30 q_2=0.38,q2*=0.30 e= ----,e*= ---- π_1=0.13, π_2=0.17 p= 0.59,p*=0.66	**W=0.36,W*=0.23** t =----,t_1=----- t*= 0.45,t^*_1=0.45 q_1=0.15,q1*=0.30 q_2=0.15,q2*=0.30 e =----,e*=---- π_1=0.11, π_2=0.11 p =0.40,p*=0.70	**W=0.41,W*=0.36** t = ----, t_1 = ----- t*= -0.81, t^*_1= --- q_1=0.81,q_1*=0.30 q_2 = 0, q2*=0.30 e = ---,e*=0.81 π_1=0.09, π_2=0.09 p=0.40,p*=0.10	**W=0.36,W*=0.41** t=---,t_1 =---- t*=-0.23,t_1*=-0.68 q_1=0.23,q1*=0.30 q_2=0.68,q2*=0.30 e =--- ,e*=0.05 π_1=0.09, π_2=0.546 p =0.40,p*=0.10
z_1	**W=0.34,W*=0.36** t = ----, t_1 = 0.23 t*= -----, t^*_1= ---- q_1=0.15,q1*=0.30 q_2=0.38,q2*=0.30 e = ---,e* = --- π_1 =0.11, π_2=0.23 p=0.48,p*=0.40	**W=0.34,W=0.34** t = ---, t_1 = 0.23 t*= ----, t^*_1= 0.23 q_1=0.15,q1*=0.38 q_2=0.38,q2*=0.15 e = ---,e* = ---- π_1=0.16, π_2=0.16 p=0.48, p*=0.48	**W=0.34,W*=0.34** t = -----,t_1=0.23 t*=0.23, t^*_1= --- q_1=0.15,q1*=0.15 q_2=0.38,q2*=0.38 e = ---,e*=----- π_1=0.04,π_2=0.28 p =0.48,p*=0.48	**W=0.34,W*=0.23** t=----, t_1=0.23 t*=0.45,t^*_1=0.45 q_1=0.15,q1*=0.15 q_2=0.15,q2*=0.38 e=--,e*= ---- π_1=0.04, π_2=0.16 p =0.48,p*=0.70	**W=0.34,W*=0.41** t=0.23, t_1= ---- t*=-0.81, t^*_1= ---- q_1=0.81,q1*=0.15 q_2=0, q2*=0.38 e = ---,e*= 0.81 π_1=0.02, π_2=0.14 p =0.48,p*=0.10	**W=0.34,W*=0.41** t = -----,t_1 =0.23 t*=-0.23, t^*_1=-0.68 q_1 =0.23,q1*=0.15 q_2=0.68,q2*=0.38 e =----,e*=0.05 π_1=0.02, π_2=0.596 p=0.48,p*=0.10
z_2	**W=0.34,W*=0.36** t = 0.23, t_1 = ---- t*= ----, t^*_1= ---- q_1=0.38,q1*=0.30 q_2=0.15,q2*=0.30 e = ----,e* = --- π_1=0.17,π_2=0.13 p =0.66,p*=0.59	**W=0.34,W*=0.34** t =0.23,t_1= ---- t*= ----,t^*_1=0.23 q_1=0.38,q1*=0.38 q_2=0.15,q2*=0.15 e = ---,e*= ---- π_1=0.28, π_2=0.04 p=0.48,p*=0.48	**W=0.34,W*=0.34** t = 0.23, t_1 = ---- t*= 0.23, t^*_1= ---- q_1=0.38,q1*=0.15 q_2=0.15,q2*=0.38 e = ---,e* = --- π_1=0.16, π_2=0.16 p =0.48,p*=0.48	**W=0.34,W*=0.23** t =0.23, t_1 =----- t*=0.45,t^*_1=0.45 q_1=0.15,q1*=0.38 q_2=0.15,q2*=0.15 e = ---,e*= ---- π_1=0.16, π_2=0.04 p =0.48,p*=0.70	**W=0.34,W*=0.41** t=0.23, t_1= ---- t*=-0.81,t^*_1= --- q_1=0.81,q1*=0.38 q_2=0, q2* = 0.15 e= ---, e*= 0.81 π_1=0.14, π_2=0.02 p =0.48,p*=0.10	**W=0.34,W*=0.41** t =0.23,t_1 =---- t*=-0.23,t^*_1=-0.68 q_1 =0.23,q1*=0.38 q_2=0.68,q2*=0.15 e =----,e*=0.05 π_1=0.14,π_2=0.478 p=0.48,p*=0.10
z_3	**W=0.23,W*=0.36** t =0.45, t_1=0.45 t*= ---, t^*_1= ---- q_1=0.15,q1*=0.30 q_2=0.15,q2*=0.30 e = ---,e* = --- π_1=0.11,π_2=0.11 p =0.70,p*=0.40	**W=0.23,W*=0.34** t =0.45, t_1 =0.45 t*= ---, t^*_1= 0.23 q_1=0.15,q1*=0.38 q_2=0.15,q2*=0.15 e= ---,e*=----- π_1=0.16, π_2=0.04 p=0.70,p*=0.48	**W=0.23,W*=0.34** t =0.45,t_1 =0.45 t*=0.23, t^*_1= ----- q_1=0.15,q1*=0.15 q_2=0.15,q2*=0.38 e = ---,e*= ---- π_1=0.04,π_2=0.16 p=0.70,p*=0.48	**W=0.23,W*=0.23** t =0.45,t_1=0.45 t*=0.45, t^*_1=0.45 q_1=0.15,q1*=0.15 q_2=0.15,q2*=0.15 e = ---,e*= ---- π_1=0.04,π_2=0.04 p=0.70,p*=0.70	**W=0.23,W*=0.41** t =0.45, t_1=0.45 t*= -0.81,t^*_1=---- q_1=0.81,q1*=0.15 q_2=0, q2*=0.15 e = ----, e*= 0.81 π_1=0.02, π_2=0.02 p=0.70,p*=0.10	**W=0.135,W*=0.41** t=0.45,t_1 =0.45 t*= -0.23,t^*_1=-0.68 q_1 =0.23,q1* =0.15 q_2=0.68,q2* =0.15 e= ---,e*= 0.05 π_1=0.23, π_2=0.478 p=0.70,p*=0.10

z_4	**W=0.41,W*=0.36** t = -0.81, t_1 = --- t*= ----, t^*_1= ---- q_1 =0, q1*=0.30 q_2=0.81,q2*=0.30 e =0.81,e*= ---- π_1=0.09,π_2=0.09 p =0.10,p*=0.40	**W=0.41,W*=0.34** t=-0.81, t_1 =---- t*=0.23, t^*_1=--- q_1=0,q1*=0.38 q_2=0.81,q2*=0.15 e =0.81,e*=---- π_1=0.14, π_2=0.02 p =0.10,p*=0.48	**W=0.41,W*=0.34** t=-0.81,t_1=--- t*=0.23,t^*_1=---- q_1=0, q1*=0.15 q_2=0.81,q2*=0.38 e =0.81,e*= --- π_1=0.02, π_2=0.14 p =0.10,p*=0.48	**W=0.41,W*=0.23** t=-0.81,t_1=---- t*=0.45,t^*_1= 0.45 q_1=0, q1* =0.15 q_2=0.81,q2*=0.15 e =0.81,e*=---- π_1=0.02, π_2=0.02 p=0.10,p*=0.70	**W=0.41,W*=0.41** t=-0.90, t_1=---- t*=-0.90,t^*_1=--- q_1= 0,q1* =0.9 q_2=0.9,q2* =0 e =0.81,e*=0.81 π_1=0, π_2=0 p=0.10,p*=0.10	**W=0.41,W*=0.41** t = -0.9,t_1 = ---- t*=-0.23,t^*_1=-0.68 q_1 =0.23,q1* =0 q_2=0.68,q2*=0.9 e =0.81,e*=0.05 π_1=0, π_2=0.45 p=0.10,p*=0.10
z_5	**W=0.41,W*=0.36** t =-0.23,t_1=-0.68 t*= ---, t^*_1=--- q_1=0.68,q1*=0.30 q_2=0.23,q2*=0.30 e = 0.05,e* = ---- π_1=0.546,π_2=0.09 p= 0.10,p*=0.40	**W=0.41,W*=0.34** t =-0.23,t_1 =-0.68 t*=-----,t^*_1=0.23 q_1=0.68,q_1*=0.38 q_2=0.23,q2*=0.15 e =0.05,e*=---- π_1=0.596, π_1 =0.02 p=0.10,p* =0.48	**W=0.41,W*=0.34** t=-0.23,t_1 =-0.68 t*=0.23,t^*_1=---- q_1=0.68,q1*=0.15 q_2=0.23,q2*=0.38 e =0.05,e*= ---- π_1=0.478,π_2=0.141 p =0.10,p*=0.48	**W=0.41,W*=0.14** t=-0.23,t_1=-0.68 t*=0.45,t^*_1=0.45 q_1=0.68,q1*=0.15 q_2=0.23,q2*=0.15 e =0.05,e*= --- π_1=0.478,π_2=0.23 p =0.10,p*=0.70	**W=0.41,W*=0.41** t=-0.23,t_1=-0.68 t*=-0.9,t^*_1=---- q_1 =0.68,q1*=0.9 q_2=0.23,q2*=0 e =0.05,e*=0.81 π_1=0.45, π_2=0 p=0.10,p*=0.10	**W=0.41,W*=0.41** t=-0.23,t_1 =-0.68 t*=-0.23,t^*_1=-0.68 q_1 =0.68,q1*=0.23 q_2=0.23,q2*=0.68 e =0.05,e*=0.05 π_1=0.45, π_2=0.45 p=0.10,p* =0.10

Табл. 6.2

	$\tilde{z}_0$	$\tilde{z}_1$	$\tilde{z}_2$	$\tilde{z}_3$	$\tilde{z}_4$	$\tilde{z}_5$
z_0	**W=0.28,W*=0.28** t = -----, t_1 = ---- t*= -----, t^*_1= ---- q_1=0.20,q1*=0.20 q_2=0.20,q2*=0.20 e = ---,e* = --- π_1=0.16,π_2=0.16 p=0.60,p*=0.60	**W=0.29,W*=0.25** t = -----, t_1 = ---- t*= -----, t^*_1=0.21 q_1=0.24,q1*=0.17 q_2=0.10,q2*=0.24 e = ---,e* = ---- π_1=0.17, π_2=0.13 p =0.59,p*=0.66	**W=0.28,W*=0.25** t = ----, t_1 = ---- t*= 0.21, t^*_1=---- q_1=0.10,q1*=0.24 q_2=0.24,q2*=0.17 e= ----,e*= ---- π_1=0.13, π_2=0.17 p= 0.59,p*=0.66	**W=0.30,W*=0.17** t =----,t_1=----- t*= 0.37,t^*_1=0.37 q_1=0.01,q1*=0.23 q_2=0.10,q2*=0.23 e =----,e*=---- π_1=0.11, π_2=0.11 p =0.55,p*=0.80	**W=0.27,W*=0.30** t = ----, t_1 = ----- t*= -0.32, t^*_1= --- q_1=0.14,q_1*=0.24 q_2=0.35, q2*=0.14 e = ---,e*=0.23 π_1=0.15, π_2=0.03 p=0.51,p*=0.62	**W=0.25,W*=0.33** t=---,t_1 =---- t*=-0.50,t_1*=-0.50 q_1=0.33,q1*=0.17 q_2=0.33,q2*=0.17 e =--- ,e*=0.22 π_1=0.04, π_2=0.26 p =0.66,p*=0.33
z_1	**W=0.25,W*=0.29** t = ----, t_1 = 0.21 t*= -----, t^*_1= ---- q_1=0.10,q1*=0.24 q_2=0.24,q2*=0.17 e = ---,e* = --- π_1 =0.13, π_2=0.17 p=0.66,p*=0.59	**W=0.26,W=0.26** t = ---, t_1 = 0.19 t*= ----, t^*_1= 0.19 q_1=0.09,q1*=0.26 q_2=0.28,q2*=0.09 e = ---,e* = ---- π_1=0.15, π_2=0.15 p=0.64, p*=0.64	**W=0.25,W*=0.25** t = -----,t_1=0.27 t*=0.27, t^*_1= --- q_1=0.13,q1*=0.13 q_2=0.22,q2*=0.22 e = ---,e*=----- π_1=0.06,π_2=0.19 p =0.65,p*=0.65	**W=0.27,W*=0.18** t=----, t_1=0.25 t*=0.42,t^*_1=0.36 q_1=0.13,q1*=0.12 q_2=0.08,q2*=0.27 e=--,e*= ---- π_1=0.06, π_2=0.14 p =0.62,p*=0.79	**W=0.24,W*=0.29** t=0.16, t_1= ---- t*=-0.26, t^*_1= ---- q_1=0.35,q1*=0.07 q_2=0.13,q2*=0.27 e = ---,e*= 0.21 π_1=0.01, π_2=0.16 p =0.66,p*=0.52	**W=0.22,W*=0.33** t = -----,t_1 =0.23 t*=-0.43, t^*_1=-0.50 q_1 =0.33,q1*=0.09 q_2=0.33,q2*=0.19 e =----,e*=0.28 π_1=0.01, π_2=0.29 p=0.72,p*=0.33
z_2	**W=0.25,W*=0.29** t = 0.21, t_1 = ---- t*= ----, t^*_1= ---- q_1=0.24,q1*=0.17 q_2=0.10,q2*=0.24 e = ----,e* = --- π_1=0.17,π_2=0.13 p =0.66,p*=0.59	**W=0.25,W*=0.25** t =0.27,t_1= ---- t*= ----,t^*_1=0.27 q_1=0.22,q1*=0.22 q_2=0.13,q2*=0.13 e = ---,e*= ---- π_1=0.19, π_2=0.06 p=0.65,p*=0.65	**W=0.25,W*=0.25** t = 0.19, t_1 = ---- t*= 0.19, t^*_1= ---- q_1=0.28,q1*=0.09 q_2=0.09,q2*=0.28 e = ---,e* = --- π_1=0.16, π_2=0.16 p =0.48,p*=0.48	**W=0.27,W*=0.18** t =0.25, t_1 =----- t*=0.36,t^*_1=0.42 q_1=0.08,q1*=0.27 q_2=0.13,q2*=0.12 e = ---,e*= ---- π_1=0.14, π_2=0.06 p =0.62,p*=0.79	**W=0.23,W*=0.31** t=0.27, t_1= ---- t*=-0.39,t^*_1= --- q_1=0.34,q1*=0.18 q_2=0.18,q2*=0.13 e= ---, e*= 0.24 π_1=0.05, π_2=0.09 p =0.70,p*=0.48	**W=0.22,W*=0.33** t =0.20,t_1 =---- t*=-0.52,t^*_1=-0.43 q_1 =0.33,q1*=0.19 q_2=0.33,q2*=0.09 e =----,e*=0.23 π_1=0.05,π_2=0.21 p=0.71,p*=0.33
z_3	**W=0.17,W*=0.30** t =0.37, t_1=0.37 t*= ---, t^*_1= ---- q_1=0.10,q1*=0.23 q_2=0.01,q2*=0.23 e = ---,e* = --- π_1=0.11,π_2=0.11	**W=0.18,W*=0.27** t =0.42, t_1 =0.36 t*= ---, t^*_1= 0.25 q_1=0.08,q1*=0.27 q_2=0.13,q2*=0.12 e= ---,e*=----- π_1=0.14, π_2=0.06	**W=0.18,W*=0.27** t =0.36,t_1 =0.42 t*=0.25, t^*_1= ----- q_1=0.13,q1*=0.12 q_2=0.08,q2*=0.27 e = ---,e*= ---- π_1=0.06,π_2=0.14	**W=0.19,W*=0.19** t =0.43,t_1=0.43 t*=0.43, t^*_1=0.43 q_1=0.11,q1*=0.11 q_2=0.11,q2*=0.11 e = ---,e*= ---- π_1=0.05,π_2=0.05	**W=0.16,W*=0.31** t =0.39, t_1=0.32 t*= -0.29,t^*_1=---- q_1=0.35,q1*=0.07 q_2=0.18,q2*=0.12 e = ----, e*= 0.21 π_1=0.01, π_2=0.09	**W=0.15,W*=0.33** t=0.32,t_1 =0.32 t*= -0.42,t^*_1=-0.42 q_1 =0.33,q1* =0.09 q_2=0.33,q2* =0.09 e= ---,e*= 0.19 π_1=0.01, π_2=0.21

	p =0.80,p*=0.55	p=0.79,p*=0.62	p=0.79,p*=0.62	p=0.77,p*=0.77	p=0.81,p*=0.47	p=0.83,p*=0.33
z_4	**W=0.30,W*=0.27** t = -0.32, t_1 = --- t*= ----, t^*_1= ---- q_1 =0.14, q1*=0.24 q_2=0.35,q2*=0.14 e =0.23,e*= ---- π_1=0.15,π_2=0.03 p =0.51,p*=0.62	**W=0.29,W*=0.24** t=-0.26, t_1 =---- t*=0.16, t^*_1=--- q_1=0.13,q1*=0.27 q_2=0.35,q2*=0.07 e =0.21,e*=---- π_1=0.16, π_2=0.01 p =0.52,p*=0.66	**W=0.31,W*=0.23** t=-0.39,t_1 =--- t*=0.27,t^*_1=---- q_1=0.18, q1*=0.13 q_2=0.34,q2*=0.18 e =0.24,e*= --- π_1=0.09, π_2=0.05 p =0.48,p*=0.70	**W=0.31,W*=0.16** t=-0.29,t_1=---- t*=0.39,t^*_1= 0.32 q_1=0.18, q1* =0.12 q_2=0.35,q2*=0.07 e =0.21,e*=---- π_1=0.09, π_2=0.01 p=0.47,p*=0.81	**W=0.282,W*=0.282** t=-0.26, t_1=---- t*=-0.26,t^*_1=--- q_1= 0.10,q1* =0.35 q_2=0.35,q2* =0.10 e =0.22,e*=0.22 π_1=0.01, π_2=0.01 p=0.55,p*=0.55	**W=0.29,W*=0.33** t = -0.50,t_1 = ---- t*=-0.44,t^*_1=-0.69 q_1 =0.33,q1* =0.10 q_2=0.33,q2*=0.35 e =0.31,e*=0.20 π_1=0.02, π_2=0.17 p=0.54,p*=0.33
z_5	**W=0.33,W*=0.25** t =-0.50,t_1=-0.50 t*= ---, t^*_1=--- q_1=0.33,q1*=0.17 q_2=0.33,q2*=0.17 e = 0.22,e* = ---- π_1=0.26,π_2=0.04 p= 0.33,p*=0.66	**W=0.33,W*=0.22** t =-0.43,t_1 =-0.52 t*=-----,t^*_1=-0.20 q_1=0.33,q_1*=0.19 q_2=0.33,q2*=0.09 e =0.28,e*=---- π_1=0.29, π_1 =0.01 p=0.33,p* =0.72	**W=0.33,W*=0.22** t=-0.52,t_1 =-0.43 t*=0.20,t^*_1=---- q_1=0.33,q1*=0.09 q_2=0.33,q2*=0.19 e =0.23,e*= ---- π_1=0.21,π_2=0.05 p =0.33,p*=0.71	**W=0.33,W*=0.15** t=-0.42,t_1=-0.42 t*=0.32,t^*_1=0.32 q_1=0.33,q1*=0.09 q_2=0.33,q2*=0.09 e =0.19,e*= --- π_1=0.21,π_2=0.01 p =0.33,p*=0.83	**W=0.33,W*=0.29** t=-0.44,t_1=-0.69 t*=-0.50,t^*_1=---- q_1 =0.33,q1*=0.35 q_2=0.33,q2*=0.10 e =0.20,e*=0.31 π_1=0.17, π_2=0.02 p=0.33,p*=0.54	**W=0.33,W*=0.33** t=-0.67,t_1 =-0.67 t*=-0.67,t^*_1=-0.67 q_1 =0.33,q1*=0.33 q_2=0.33,q2*=0.33 e =0.28,e*=0.28 π_1=0.17, π_2=0.17 p=0.33,p* =0.33

Proof of Theorem 4.1.:

I) Taking into account the imposing of quota by q_1 and v_1 we get that $q_1 \in [0,\overline{q}], v_1 \in [0,\overline{v}]$. Thus maximization π_1 by q_1 can be considered on a compact set $X_3 = [0,\overline{q}]$, and maximization π_1^* by v_1 - on a compact set $X_4 = [0,\overline{v}]$.

II) By the data 1),2) functions $\pi_1 (q_1, v_1)$ and $\pi_1^*(q_1, v_1)$ are continuous.

III) As according to the conditions 1) - 3) the theorems guarantee the following conditions:

$$\frac{\partial^2 \pi_1}{\partial q_1^2} = p''\cdot q_1 + 2\cdot p' - c'' < 0, \qquad \frac{\partial^2 \pi_1^*}{\partial v_1^2} = p''\cdot v_1 + 2\cdot p' - c^{*}{}'' < 0 \qquad (4.13)$$

The conditions (4.13) guarantee a concavity of functions $\pi_1 (q_1, v_1)$ and $\pi_1^*(q_1, v_1)$.

IV) Thus from I), II) and III) on the basis of the theorem of Nash it follows that there is an Curnout equilibrium on the second step of the game $\left\langle q_1^0(t,t^*), v_1^0(t,t^*)\right\rangle$.

Now we shall return to the first step of the game and we shall consider the game between governments. Let's show that the conditions of the theorem guarantee the existence of Nash equilibrium on the first step as well, an consequently the existence of the perfect subgame Nash equilibrium, which will make the optimum two-part trade policy.

V) In the beginning we shall show that the problem of maximization $f_1(x)$ by t can be considered on a segment (compact set) X1=$[t_н, t_в]$ and the problem of maximization $f_2(x)$ by t* can be considered on a segment (compact set) X2=$[t_н^*, t_в^*]$. This fact immediately comes from continuity and limitation of functions $q_1(t,t^*), v_1(t,t^*)$.

VI) As $p(Q) \in C^2$; $c\ (q) \in C^2$ and $c^*(v) \in C^2$, then functions $f_1(t,t^*) = p \cdot q_1(t,t^*) - c\ (q_1(t,t^*))$; $f_2(t,t^*) = p\ \cdot v_1(t,t^*) - c^*(v_1(t,t^*))$ are continuous.

VII) Further we shall prove concavity of functions $f_1(t,t^*)$ and $f_2(t,t^*)$ by t и t* accordingly.

We differentiate function $f_1(t,t^*)$ twice by t :

$$\frac{\partial^2 f_1(t,t^*)}{\partial t^2} = (p''\cdot q_1 + 2p' - c'')\cdot\left(\frac{\partial q_1}{\partial t}\right)^2 + 2(p''\cdot q_1 + p')\cdot\frac{\partial q_1}{\partial t}\cdot\frac{\partial v_1}{\partial t} + p''\cdot q_1\cdot\left(\frac{\partial v_1}{\partial t}\right)^2 \qquad (4.14)$$

We shall use the Lemma 4.1. and Corollary 4.1. And then if $p'' < 0$ then the estimation is fair

$$\frac{\partial^2 f_1(t,t^*)}{\partial t^2} = (p''\cdot v_1\cdot(\alpha+1)^2 + 2\cdot p'(1+\alpha) - c\ '')\cdot\left(\frac{\partial q_1}{\partial t}\right)^2 < 0$$

if $p'' \geq 0$ then the estimation is fair

$$\frac{\partial^2 f_1(t,t^*)}{\partial t^2} = (p''\cdot v_1\cdot(\alpha+1)^2 + 2\cdot p'(1+\alpha) - c\ '')\cdot\left(\frac{\partial q_1}{\partial t}\right)^2 \leq$$
$$\leq (p''q + p')(1+\alpha) + p'(1+\alpha) - c'' < 0$$

Thus function $f_1(t,t^*)$ is concave by t. Similarly we obtain a concavity $f_2(t,t^*)$ by t*.

VIII) So, from V), VI) and VII) On the basis of the Nash theorem it follows that there is Nash equilibrium on the first step of the game (t^0, t^{*0}). From IV) and VIII) it follows that in the game Γ_1 there exists perfect subgame Nash equilibrium: $X^0 = \left\langle (e^0, \overline{q}^0, t^0), (e^{*0}, \overline{v}^0, t^{*0}), (q_1^0(t,t^*), (v_1^0(t,t^*) \right\rangle$, where

$$e^{*0} = p(q_1^0(t^0,t^{*0}) + v_1^0(t,t^{*0})\cdot q_1^0(t^0,t^{*0}) - c\ (q_1^0(t^0,t^{*0})) - t^0\cdot q_1^0(t^0,t^{*0}),$$

$$e^0 = p(q_1^0(t^0,t^{*0}) + v_1^0(t^0,t^{*0}))\cdot v_1^0(t^0,t^{*0}) - c^*(v_1^0(t^0,t^{*0})) - t^0\cdot v_1^0(t^0,t^{*0}),$$

$\bar{v}^0 = v_1^0(t^0, t^{*0})$, $\bar{q}^0 = q_1^0(t^0, t^{*0})$. Proved.

Proof of Corollary 4.2.:

As the existence of the optimum policy is proved in the theorem now it is possible to return to a problem of its searching. Differentiating functions $f_1(t,t^*)$ by t , we get:

$$\frac{\partial\ f_1(t,t^*)}{\partial t} = (p' \cdot q_1 + p - c\ ') \cdot \frac{\partial q_1}{\partial t} + p' \cdot q_1 \cdot \frac{\partial v_1}{\partial t};\ .$$

From conditions (4.7) we have $t = p' q_1 + p - c'$ then

$$\frac{\partial\ f_1(t,t^*)}{\partial t} = t \cdot \frac{\partial q_1}{\partial t} + p' \cdot q_1 \cdot \frac{\partial v_1}{\partial t}; \qquad (4.15)$$

From (4.15), Lemma 4.1. condition 2) Theorem 4.1. we get that

$$\left.\frac{\partial\ f_1(t,t^*)}{\partial t}\right|_{t=0} < 0; \qquad (4.16)$$

Similarly,

$$\left.\frac{\partial\ f_2(t,t^*)}{\partial t^*}\right|_{t^*=0} < 0; \qquad (4.17)$$

The conditions (4.16) and (4.17) imply that the optimum two-part tariff is a subsidy. Thus the optimum payment for the license is equal to flowing sales proceeds of a internal firm: e^0, e^{*0}

Proof of Lemma 4.2.:

Because of homogeneity and symmetry, the N – models of home firms are identical, as well as N* of foreign firms. That's why total sales in the two countries equal to Q and Q*.

$Q = N \cdot q_1 + N^* \cdot v_1$ (4.19) $\qquad Q = N \cdot q_1^* + N^* \cdot v_1^*$ (4.20)

From the first-order conditions in the Coumot model for home and foreign firms we have the followingconditions:

$$\begin{cases} \dfrac{\partial \pi_1}{\partial q_1} = p' \cdot q_1 + p - c\ ' = 0 \\ \dfrac{\partial \pi_1}{\partial q_1^*} = p^*{}' \cdot q_1^* + p^* - c\ ' - t^* = 0 \\ \dfrac{\partial \pi_1^*}{\partial v_1} = p' \cdot v_1 + p - c^*{}' - t = 0 \\ \dfrac{\partial \pi_1^*}{\partial v_1^*} = p^*{}' \cdot v_1^* + p^* - c^* = 0 \end{cases} \qquad (4.21)$$

The second-order conditions are executed here under the following virtue of conditions 1) and 2) Lemma 4.2, as they guarantee that

$$\begin{aligned} &\frac{\partial^2 \pi_1}{\partial q_1^2} = p'' \cdot q_1 + 2 \cdot p' - c\ '' < 0, && \frac{\partial^2 \pi_1^*}{\partial v_1^2} = p'' \cdot v_1 + 2 \cdot p' - c^*{}'' < 0 \\ &\frac{\partial^2 \pi_1}{\partial q_1 q_1^*} = -c\ '' < 0, && \frac{\partial^2 \pi_1^*}{\partial v_1 v_1^*} = -c^*{}'' < 0 \qquad (4.22) \\ &\frac{\partial \pi_1}{\partial q_1^{*2}} = p^*{}'' \cdot q_1^* + 2 \cdot p^*{}' - c\ '' < 0 && \frac{\partial \pi_1^*}{\partial v_1^{*2}} = p^*{}'' \cdot v_1^* + 2 \cdot p^*{}' - c^*{}'' < 0 \end{aligned}$$

The decision of the first-order conditions (4.21) will give q_1, q_1^*, v_1, v_1^*, as functions t and t*. This decision has tendency, that the firms will sell their product both on internal and external market. The comparative static effects $\frac{\partial q_1}{\partial t}, \frac{\partial v_1}{\partial t}, \frac{\partial q_1^*}{\partial t}, \frac{\partial v_1^*}{\partial t}, \frac{\partial v_1^*}{\partial t^*}, \frac{\partial q_1^*}{\partial t^*}, \frac{\partial v_1^*}{\partial t^*}, \frac{\partial q_1}{\partial t^*}$ can be obtained at total differentiating (4.21) with respect to t, t* and the levels of production of the firms. For the home market we have the system:

$$\begin{cases} \left(N \cdot (p'' \cdot q_1 + p') + p' - c\ ''\right) \cdot \dfrac{\partial q_1}{\partial t} - c\ '' \cdot \dfrac{\partial q_1^*}{\partial t} + N^* \cdot (p'' \cdot q_1 + p') \cdot \dfrac{\partial v_1}{\partial t} = 0 \\ -c'' \cdot \dfrac{\partial q_1}{\partial t} + \left(N\ \ \cdot (p^*{}'' \cdot q_1^* + p^*{}') + p^*{}' - c\ ''\right) \dfrac{\partial q_1^*}{\partial t} + N^* \cdot (p^*{}'' \cdot q_1^* + p^*{}') \cdot \dfrac{\partial v_1^*}{\partial t} = 0 \\ N \cdot (p'' \cdot v_1 + p') \cdot \dfrac{\partial q_1}{\partial t} + \left(N^* \cdot (p'' \cdot v_1 + p') + p' - c^*{}''\right) \cdot \dfrac{\partial v_1}{\partial t} - c_1^* \cdot \dfrac{\partial v_1^*}{\partial t} = 1 \\ N\ \ \cdot (p^*{}'' \cdot v_1^* + p^*{}') \cdot \dfrac{\partial q_1^*}{\partial t} - c^*{}'' \cdot \dfrac{\partial v_1}{\partial t} + \left(N^* \cdot (p^*{}'' \cdot v_1^* + p^*{}') + p^*{}' - c^*{}''\right) \cdot \dfrac{\partial v_1^*}{\partial t} = 0 \end{cases} \Leftrightarrow$$

$\Leftrightarrow$

$$\begin{cases} \dfrac{\partial q_1}{\partial t} + \dfrac{-c''}{\left(N \cdot (p''\cdot q_1 + p') + p' - c''\right)} \cdot \dfrac{\partial q_1^*}{\partial t} + \dfrac{N^* \cdot (p''\cdot q_1 + p')}{\left(N \cdot (p''\cdot q_1 + p') + p' - c''\right)} \cdot \dfrac{\partial v_1}{\partial t} = 0 \\ \cdot \dfrac{-c''}{\left(N \cdot (p^{*}{''}\cdot q_1^* + p^{*}{'}) + p^{*}{'} - c''\right)} \dfrac{\partial q_1}{\partial t} + \dfrac{\partial q_1^*}{\partial t} + \dfrac{N^* \cdot (p^{*}{''}\cdot q_1^* + p^{*}{'})}{\left(N \cdot (p^{*}{''}\cdot q_1^* + p^{*}{'}) + p^{*}{'} - c''\right)} \cdot \dfrac{\partial v_1^*}{\partial t} = 0 \\ \dfrac{N \cdot (p''\cdot v_1 + p')}{\left(N^* \cdot (p''\cdot v_1 + p') + p' - c^{*}{''}\right)} \cdot \dfrac{\partial q_1}{\partial t} + \dfrac{\partial v_1}{\partial t} + \dfrac{-c_1^*}{\left(N^* \cdot (p''\cdot v_1 + p') + p' - c^{*}{''}\right)} \cdot \dfrac{\partial v_1^*}{\partial t} = \dfrac{1}{\left(N^* \cdot (p''\cdot v_1 + p') + p' - c^{*}{''}\right)} \\ \dfrac{N \cdot (p^{*}{''}\cdot v_1^* + p^{*}{'})}{\left(N^* \cdot (p^{*}{''}\cdot v_1^* + p^{*}{'}) + p^{*}{'} - c^{*}{''}\right)} \cdot \dfrac{\partial q_1^*}{\partial t} + \dfrac{-c^{*}{''}}{\left(N^* \cdot (p^{*}{''}\cdot v_1^* + p^{*}{'}) + p^{*}{'} - c^{*}{''}\right)} \cdot \dfrac{\partial v_1}{\partial t} + \dfrac{\partial v_1^*}{\partial t} = 0 \end{cases} \tag{4.23}$$

We will introduce enter designations, taking into account, that according to the condition of Lemma 4.2. N=N*:

$$\begin{cases} g_1 = \dfrac{-c''}{\left(N \cdot (p''\cdot q_1 + p') + p' - c''\right)}; g_2 = \dfrac{N \cdot (p''\cdot q_1 + p')}{\left(N \cdot (p''\cdot q_1 + p') + p' - c''\right)} \\ g_3 = \dfrac{-c''}{\left(N \cdot (p^{*}{''}\cdot q_1^* + p^{*}{'}) + p^{*}{'} - c''\right)}; g_4 = \dfrac{N \cdot (p^{*}{''}\cdot q_1^* + p^{*}{'})}{\left(N \cdot (p^{*}{''}\cdot q_1^* + p^{*}{'}) + p^{*}{'} - c''\right)} \\ g_5 = \dfrac{N \cdot (p''\cdot v_1 + p')}{\left(N \cdot (p''\cdot v_1 + p') + p' - c^{*}{''}\right)}; g_6 = \dfrac{-c_1^*}{\left(N \cdot (p''\cdot v_1 + p') + p' - c^{*}{''}\right)}; g = \dfrac{1}{\left(N \cdot (p''\cdot v_1 + p') + p' - c^{*}{''}\right)} \\ g_7 = \dfrac{N \cdot (p^{*}{''}\cdot v_1^* + p^{*}{'})}{\left(N \cdot (p^{*}{''}\cdot v_1^* + p^{*}{'}) + p^{*}{'} - c^{*}{''}\right)}; g_8 = \dfrac{-c^{*}{''}}{\left(N \cdot (p^{*}{''}\cdot v_1^* + p^{*}{'}) + p^{*}{'} - c^{*}{''}\right)} \end{cases}$$

From conditions 1) - 4) of Lemma 4.2 it is obvious, that

$$g<0;\ 0 < g_k < 1, \forall k = 1,...8; \quad g_1 + g_2 \le a_1\ ;\ g_3 + g_4 \le a_2\ ;$$

$$g_5 + g_6 \le a_3;\ g_7 + g_8 \le a_4\ , 0 < a_i < 1, i = 1,..,4 \tag{4.24}$$

In new designations the determinant of a matrix of the left part of the system (4.23) :

$$D(g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8) = \begin{vmatrix} 1 & g_1 & g_2 & 0 \\ g_3 & 1 & 0 & g_4 \\ g_5 & 0 & 1 & g_6 \\ 0 & g_7 & g_8 & 1 \end{vmatrix} =$$

$$= 1 - (g_1 g_6 + g_4 g_7 + g_1 g_3 + g_2 g_5) - (g_2 g_3 g_6 g_7 + g_1 g_4 g_5 g_8) + (g_1 g_3 g_6 g_8 + g_2 g_4 g_5 g_7)$$

We'll prove that $D(g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8) > 0, \forall g_k$. In the beginning we shall show that the determinant can obtain the extreme value only on the border. It is really so because of continuity

$D(g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8)$ and because of the fact that $\frac{\partial D}{\partial g_k}$ does not depend on g_k . Moreover

$\frac{\partial D}{\partial g_k} < 0$:

$$\frac{\partial D}{\partial g_1} = g_3 \cdot (g_6 g_8 - 1) - g_4 g_5 g_8 < 0; \ \frac{\partial D}{\partial g_2} = g_5 \cdot (g_4 g_7 - 1) - g_3 g_6 g_7 < 0; \ \frac{\partial D}{\partial g_3} = g_1 \cdot (g_6 g_8 - 1) - g_2 g_6 g_7 < 0;$$

$$\frac{\partial D}{\partial g_4} = g_7 \cdot (g_2 g_5 - 1) - g_1 g_5 g_8 < 0; \ \frac{\partial D}{\partial g_5} = g_2 \cdot (g_4 g_7 - 1) - g_1 g_4 g_8 < 0;$$

$$\frac{\partial D}{\partial g_6} = g_8 \cdot (g_1 g_3 - 1) - g_2 g_3 g_7 < 0; \ \frac{\partial D}{\partial g_7} = g_4 \cdot (g_2 g_5 - 1) - g_2 g_3 g_6 < 0;$$

$$\frac{\partial D}{\partial g_8} = g_6 \cdot (g_1 g_3 - 1) - g_1 g_4 g_5 < 0;$$

From this it follows that the minimum value is reached by D only on the right border, i.e.

$$D \geq \min_{\substack{g_k: 0<g_k<1, \forall k=1,\dots 8; \\ g_1+g_2 \leq a_1; g_3+g_4 \leq a_2; g_5+g_6 \leq a_3; g_7+g_8 \leq a_4}} (D) \geq \min_{\substack{g_1, g_3, g_5, g_7: 0<g_k<1, \forall k=1,3,5,7; \\ g_2=a_1-g_1; g_4=a_2-g_3; g_6=a_3-g_5; g_8=a_4-g_7 \\ 0<a_i<1, \forall i=1,..4}} (D) \geq \min_{\substack{g_1, g_3, g_5, g_7: 0<g_k<1, \forall k=1,3,5,7; \\ g_2=a-g_1; g_4=a-g_3; g_6=a-g_5; g_8=a-g_7 \\ 0<a<1}} (D) =$$

$$= \min_{\substack{g_1, g_3, g_5, g_7: \\ 0<g_k<1, \forall k=1,3,5,7; 0<a<1}} \left((a-1) \cdot (a+1) \cdot \left[(g_3 - g_5) \cdot (g_1 - g_7) - 1 \right] \right) > 0$$

as $0 < a < 1$; $-1 < g_3 - g_5 < 1$; $-1 < g_1 - g_7 < 1$; (here $a = \max\{a_1, a_2, a_3, a_4\}$).

It is proved D>0.

It allows to assert that the system (4.23) has the sole decision:

$\frac{\partial q_1}{\partial t} = \frac{D_1}{D}; \frac{\partial q_1^*}{\partial t} = \frac{D_2}{D}; \frac{\partial v_1}{\partial t} = \frac{D_3}{D}; \frac{\partial v_1^*}{\partial t} = \frac{D_4}{D}$, where

$$D_1 = \begin{vmatrix} 0 & g_1 & g_2 & 0 \\ 0 & 1 & 0 & g_4 \\ g & 0 & 1 & g_6 \\ 0 & g_7 & g_8 & 1 \end{vmatrix}; D_2 = \begin{vmatrix} 1 & 0 & g_2 & 0 \\ g_3 & 0 & 0 & g_4 \\ g_5 & g & 1 & g_6 \\ 0 & 0 & g_8 & 1 \end{vmatrix}; D_3 = \begin{vmatrix} 1 & g_1 & 0 & 0 \\ g_3 & 1 & 0 & g_4 \\ g_5 & 0 & g & g_6 \\ 0 & g_7 & 0 & 1 \end{vmatrix}; \ D_1 = \begin{vmatrix} 1 & g_1 & g_2 & 0 \\ g_3 & 1 & 0 & 0 \\ g_5 & 0 & 1 & g \\ 0 & g_7 & g_8 & 0 \end{vmatrix}.$$

From the conditions (4.24) we get:

$$D_1 = -g(g_2 \cdot (1 - g_4 g_7) + g_1 g_4 g_8) > 0; D_2 = g(g_4 g_8 + g_3 g_2) < 0;$$

$$D_3 = -g(g_4 g_7 + g_1 g_3 - 1) < 0; D_4 = g(g_8 \cdot (g_1 g_3 - 1) - g_2 g_3 g_7) > 0.$$

Thus, $\frac{\partial q_1}{\partial t} > 0; \frac{\partial q_1^*}{\partial t} < 0; \frac{\partial v_1}{\partial t} < 0; \frac{\partial v_1^*}{\partial t} > 0.$

The similar calculations give conditions: $\frac{\partial v_1^*}{\partial t^*} > 0; \frac{\partial v_1}{\partial t^*} < 0; \frac{\partial q_1^*}{\partial t^*} < 0; \frac{\partial q_1}{\partial t^*} > 0.$

Proof of Corollary 4.3.:

From the proved Lemma 4.2 the equality is fair:

$$\frac{\partial q_1}{\partial t} \Big/ \frac{\partial v_1}{\partial t} = D_1 / D_3 = -\frac{g_2(1-g_4g_7)+g_1g_4g_8}{1-(g_4g_7+g_1g_3)} < 0 .$$

In order to show that $\frac{\partial q_1}{\partial t} \Big/ \frac{\partial v_1}{\partial t} > -1$ It is enough to show that

$$1-(g_4g_7+g_1g_3) > g_2(1-g_4g_7)+g_1g_4g_8 \Leftrightarrow 1-(g_4g_7+g_1g_3)-g_2(1-g_4g_7)-g_1g_4g_8 > 0$$

From (4.24) we get,

$$1-(g_4g_7+g_1g_3)-g_2(1-g_4g_7)-g_1g_4g_8 \geq$$
$$\geq 1-a_1-a_2g_7+g_7g_3-g_1g_3+a_1a_2g_7-a_1g_7g_3+g_1-a_2a_4g_1+a_4g_1g_3 \geq (1-a)\cdot(1-g_4g_7+g_1+g_1g_4) > 0$$

(here $a = \max\{a_1, a_2, a_3, a_4\}$).

The last estimation proves that $\frac{\partial q_1}{\partial t} = \alpha \frac{\partial v_1}{\partial t}$, where $\alpha \in (-1,0)$.

It is similarly proved that $\frac{\partial v_1^*}{\partial t^*} = \beta \frac{\partial q_1^*}{\partial t^*}$, where $\beta \in (-1,0)$. Proved.

Proof of Theorem 4.2.: As the proof in the most general case is very voluminous, we shall prove the given theorem for the case N=1 (one home firm) and N * = 1 (one foreign firm).

I) In the beginning we shall prove, that for the given two-part politics $x_1 = (e, \tilde{v}, t) \in X_1$, $x_2 = (e^*, \tilde{q}, t^*) \in X_2$ home and foreign governments there is Nash equilibrium on the second step when the firms play an equilibrium Curnout and then we shall consider Nash equilibrium between governments of the two countries on a first step. Let's remark that

$\frac{\partial \pi_1}{\partial q_1} = p' \cdot q_1 + p - c\,' < 0$, for $q_1 \geq \hat{Q}$, as $p'(Q) < 0$ (by the data 2)) and $p(q_1) < c_1'(q_1)$, under $q_1 \geq \hat{Q}$ (by the data 5)). Then profit in a point $\hat{Q}$ bigger than in any point $q_1 \geq \hat{Q}$. Thus maximization π_1 by q_1 under $q_1 \geq 0$ is reduced to a maximization on a segment $[0, \hat{Q}]$. Similarly we obtain that a maximization π_1^* by v_1^* under $v_1^* \geq 0$ is reduced to a maximization on a segment $[0, \hat{Q}^*]$. Then taking into account the imposing of quota by q_1^* and v_1 we get that $q_1^* \in [0, \tilde{q}], v_1 \in [0, \tilde{v}]$. Thus maximization $\pi_1(q_1, q_1^*)$ by (q_1, q_1^*) can be considered on a compact set $X_3 = [0, \hat{Q}] \times [0, \tilde{q}]$, and maximization $\pi_1^*(v_1, v_1^*)$ by (v_1, v_1^*) - on a compact set $X_4 = [0, \tilde{v}] \times [0, \hat{Q}]$.

II) As on a condition 2) function $p(Q), p^*(Q^*) \in C^2$, by the data 1) $c\ (q) \in C^2$ and $c^*(v) \in C^2$ then functions $\pi_1(q_1, q_1^*)$ and $\pi_1^*(v_1, v_1^*)$ are continuous.

III) As according to the conditions 1) and 2) the theorems guarantee the following conditions:

$$\begin{aligned}
&\frac{\partial^2 \pi_1}{\partial q_1^2} = p'' \cdot q_1 + 2 \cdot p' - c\,'' < 0, && \frac{\partial^2 \pi_1^*}{\partial v_1^2} = p'' \cdot v_1 + 2 \cdot p' - c^*{}'' < 0 \\
&\frac{\partial^2 \pi_1}{\partial q_1 q_1^*} = -c\,'' < 0, && \frac{\partial^2 \pi_1^*}{\partial v_1 v_1^*} = -c^*{}'' < 0 \\
&\frac{\partial \pi_1}{\partial q_1^{*2}} = p^*{}'' \cdot q_1^* + 2 \cdot p^*{}' - c\,'' < 0 && \frac{\partial \pi_1^*}{\partial v_1^{*2}} = p^*{}'' \cdot v_1^* + 2 \cdot p^*{}' - c^*{}'' < 0
\end{aligned} \tag{4.27}$$

The conditions (4.27) guarantee a concavity of functions $\pi_1(q_1, q_1^*)$ and $\pi_1^*(v_1, v_1^*)$.

IV) Thus from I), II) and III) on the basis of the theorem of Nash it follows that there is an Curnout equilibrium on the second step of the game $\left\langle (q_1^0(t,t^*), q_1^{*0}(t,t^*)), (v_1^0(t,t^*), v_1^{*0}(t,t^*)) \right\rangle$.

Now we shall return to the first step of the game and we shall consider the game between governments. Let's show that the conditions of the theorem guarantee the existence of Nash equilibrium on the first step as well, an consequently the existence of the perfect subgame Nash equilibrium, which will make the optimum two-part trade policy. (It is necessary to notice, that in early works concerning the two-part trade policy the a game situation have never appeared at a level of governments, at earlier governments solved simple optimization problem, e.g. Fuerst and Kim (1997)).

V) In the beginning we shall show that the problem of maximization $G(t,t^*)$ by t can be considered on a segment (compact set) X$_1$=$[t_н,t_в]$ and the problem of maximization $G^*(t,t^*)$ by t* can be considered on a segment (compact set) X$_2$=$[t^*_н,t^*_в]$. This fact immediately comes from continuity and limitation of functions $q_1(t,t^*), q_1^*(t,t^*), v_1(t,t^*), v_1^*(t,t^*)$.

VI) As $p(Q), p^*(Q^*) \in C^2$; $c(q) \in C^2$ and $c^*(v) \in C^2$, then functions $G(t,t^*) = p \cdot v_1(t,t^*) - c^*(v_1(t,t^*))$; $G^*(t,t^*) = p^* \cdot q_1^*(t,t^*) - c(q_1^*(t,t^*))$ are continuous.

VII) Further we shall prove concavity of functions $G(t,t^*)$ and $G^*(t,t^*)$ by t и t* accordingly.

We differentiate function $G(t,t^*)$ twice by t :

$$\frac{\partial^2 G(t,t^*)}{\partial t^2} = p''v_1 \cdot \left(\frac{\partial q_1}{\partial t}\right)^2 + 2\cdot(p''v_1 + p')\cdot\frac{\partial q_1}{\partial t}\cdot\frac{\partial v_1}{\partial t} + (p''v_1 + 2p' - c'')\cdot\left(\frac{\partial v_1}{\partial t}\right)^2 \quad (4.28)$$

We shall use the Lemma 4.2 which was previously proved by author of the project:

$\frac{\partial q_1}{\partial t} > 0$, $\frac{\partial v_1}{\partial t} < 0$ и $\frac{\partial q_1}{\partial t} = \alpha\frac{\partial v_1}{\partial t}$, $\alpha \in (-1,0)$.

And then if $p'' < 0$ then the estimation is fair

$$\frac{\partial^2 G(t,t^*)}{\partial t^2} = p''v_1 \cdot \left(\frac{\partial q_1}{\partial t}\right)^2 + 2\cdot(p''v_1 + p')\cdot\frac{\partial q_1}{\partial t}\cdot\frac{\partial v_1}{\partial t} + (p''v_1 + 2p' - c'')\cdot\left(\frac{\partial v_1}{\partial t}\right)^2 =$$
$$= \left(\frac{\partial v_1}{\partial t}\right)^2 \cdot \left(p''v_1\cdot(\alpha-1)^2 + 2\cdot p'(\alpha+1) - c''\right) < 0$$

if $p'' \geq 0$ then the estimation is fair

$$\frac{\partial^2 G(t,t^*)}{\partial t^2} = p''v_1 \cdot \left(\frac{\partial q_1}{\partial t}\right)^2 + 2\cdot(p''v_1 + p')\cdot\frac{\partial q_1}{\partial t}\cdot\frac{\partial v_1}{\partial t} + (p''v_1 + 2p' - c'')\cdot\left(\frac{\partial v_1}{\partial t}\right)^2 =$$
$$= p''v_1\cdot\left(\frac{\partial q_1}{\partial t}\right)^2 - p''v_1\cdot\left(\frac{\partial v_1}{\partial t}\right)^2 + 2\cdot(p''v_1 + p')\cdot\frac{\partial q_1}{\partial t}\cdot\frac{\partial v_1}{\partial t} + 2\cdot(p''v_1 + p')\cdot\left(\frac{\partial v_1}{\partial t}\right)^2 - c''\cdot\left(\frac{\partial v_1}{\partial t}\right)^2 =$$
$$= \left(\frac{\partial v_1}{\partial t}\right)^2 \cdot \left(p''v_1\cdot(\alpha^2 - 1) + 2\cdot(p''v_1 + p')\cdot(\alpha+1) - c''\right) < 0$$

Thus function $G(t,t^*)$ is concave by t. Similarly we obtain a concavity $G^*(t,t^*)$ by t*.

VIII) So, from V), VI) and VII) On the basis of the Nash theorem it follows that there is Nash equilibrium on the first step of the game $\left(t^0, t^{*0}\right)$. From IV) and VIII) it follows that in the game Γ there exists perfect subgame Nash equilibrium: $X^0 = \left\langle (e^0, \tilde{v}^0, t^0), (e^{*0}, \tilde{q}^0, t^{*0}), (q_1^0(t,t^*), q_1^{*0}(t,t^*)), (v_1^0(t,t^*), v_1^{*0}(t,t^*)) \right\rangle$, where

$$e^{*0} = p^*(v_1^0(t^0,t^{*0}) + q_1^{*0}(t^0,t^{*0})) \cdot q_1^{*0}(t^0,t^{*0}) - c\ (q_1^{*0}(t^0,t^{*0})) - t^{*0} \cdot q_1^{*0}(t^0,t^{*0}),$$

$$e^0 = p(q_1^0(t^0,t^{*0}) + v_1^0(t^0,t^{*0})) \cdot v_1^0(t^0,t^{*0}) - c^*(v_1^0(t^0,t^{*0})) - t^0 \cdot v_1^0(t^0,t^{*0}),$$

$\tilde{v}^0 = v_1^0(t^0,t^{*0})$, $\tilde{q}^0 = q_1^{*0}(t^0,t^{*0})$. Proved.

Proof of Corollary 4.4.:

As the existence of the optimum policy is proved in the theorem now it is possible to return to a problem of its searching. Differentiating functions $G(t,t^*)$ и $G^*(t,t^*)$ by t and t* accordingly, we get:

$$\frac{\partial\ G(t,t^*)}{\partial t} = (p' v_1 + p - c^*{}') \cdot \frac{\partial v_1}{\partial t} + p' v_1 \cdot \frac{\partial q_1}{\partial t}; \quad \frac{\partial\ G^*(t,t^*)}{\partial t^*} = (p^*{}' q_1^* + p^* - c') \cdot \frac{\partial q_1^*}{\partial t^*} + p^*{}' q_1^* \cdot \frac{\partial v_1^*}{\partial t}.$$

From conditions (4.21) we have $\begin{cases} t = p' v_1 + p - c^*{}' \\ t^* = p^*{}' q_1^* + p^* - c' \end{cases}$, then

$$\frac{\partial\ G(t,t^*)}{\partial t} = t \cdot \frac{\partial v_1}{\partial t} + p' v_1 \cdot \frac{\partial q_1}{\partial t}; \qquad \frac{\partial\ G^*(t,t^*)}{\partial t^*} = t^* \cdot \frac{\partial q_1^*}{\partial t^*} + p^*{}' q_1^* \cdot \frac{\partial v_1^*}{\partial t} \tag{4.29}$$

From (4.29), Lemma 4.2 condition 2) Theorem 4.2 we get that

$$\left.\frac{\partial\ G(t,t^*)}{\partial t}\right|_{t=0} < 0; \qquad \left.\frac{\partial\ G^*(t,t^*)}{\partial t^*}\right|_{t^*=0} < 0 \tag{4.30}$$

The conditions (4.30) imply that the optimum two-part tariff is a subsidy. Thus the optimum payment for the license is equal to flowing sales proceeds of a foreign firm:

$$e^{*0} = p^*(v_1^0(t^0,t^{*0}) + q_1^{*0}(t^0,t^{*0})) \cdot q_1^{*0}(t^0,t^{*0}) - c_1(q_1^{*0}(t^0,t^{*0})) - t^{*0} \cdot q_1^{*0}(t^0,t^{*0}),$$

$$e^0 = p(q_1^0(t^0,t^{*0}) + v_1^0(t^0,t^{*0})) \cdot v_1^0(t^0,t^{*0}) - c_1^*(v_1^0(t^0,t^{*0})) - t^0 \cdot v_1^0(t^0,t^{*0})$$

Proof of Theorem 4.3: As the proof in the most general case is very volumeous, we shall prove the given theorem for the case N=1 (one home firm) and N * = 1 (one foreign firm).

I) In the beginning we shall prove, that for the given two-part politics $x_1 = (e, \tilde{v}, t) \in X_1$, $x_2 = (e^*, \tilde{q}, t^*) \in X_2$ home and foreign governments there is Nash equilibrium on the second step when the firms play an equilibrium Curnout and then we shall consider Nash equilibrium between governments of the two countries on a first step. Let's remark that

$\frac{\partial \pi_1}{\partial q_1} = p' \cdot q_1 + p - c\,' < 0$, for $q_1 \geq \hat{Q}$, as $p'(Q) < 0$ (by the data 2)) and $p(q_1) < c_1{}'(q_1)$, under $q_1 \geq \hat{Q}$ (by the data 5)). Then profit in a point $\hat{Q}$ bigger than in any point $q_1 \geq \hat{Q}$. Thus maximization π_1 by q_1 under $q_1 \geq 0$ is reduced to a maximization on a segment $[0, \hat{Q}]$. Similarly we obtain that a maximization π_1^* by v_1^* under $v_1^* \geq 0$ is reduced to a maximization on a segment $[0, \hat{Q}^*]$. Then taking into account the imposing of quota by q_1^* and v_1 we get that $q_1^* \in [0, \tilde{q}], v_1 \in [0, \tilde{v}]$. Thus maximization $\pi_1(q_1, q_1^*)$ by (q_1, q_1^*) can be considered on a compact set $X_3 = [0, \hat{Q}] \times [0, \tilde{q}]$, and maximization $\pi_1^*(v_1, v_1^*)$ by (v_1, v_1^*) - on a compact set $X_4 = [0, \tilde{v}] \times [0, \hat{Q}]$.

II) As on a condition 2) function $p(Q), p^*(Q^*) \in C^2$, by the data 1) $c\,(q) \in C^2$ and $c^*(v) \in C^2$ then functions $\pi_1(q_1, q_1^*)$ and $\pi_1^*(v_1, v_1^*)$ are continuous.

III) As according to the conditions 1) and 2) the theorems guarantee the following conditions:

$$
\begin{aligned}
&\frac{\partial^2 \pi_1}{\partial q_1^2} = p'' \cdot q_1 + 2 \cdot p' - c\,'' < 0, && \frac{\partial^2 \pi_1^*}{\partial v_1^2} = p'' \cdot v_1 + 2 \cdot p' - c^*{}'' < 0 \\
&\frac{\partial^2 \pi_1}{\partial q_1 q_1^*} = -c\,'' < 0, && \frac{\partial^2 \pi_1^*}{\partial v_1 v_1^*} = -c^*{}'' < 0 \\
&\frac{\partial \pi_1}{\partial q_1^{*2}} = p^*{}'' \cdot q_1^* + 2 \cdot p^*{}' - c\,'' < 0 && \frac{\partial \pi_1^*}{\partial v_1^{*2}} = p^*{}'' \cdot v_1^* + 2 \cdot p^*{}' - c^*{}'' < 0
\end{aligned}
\qquad (4.33)
$$

The conditions (4.33) guarantee a concavity of functions $\pi_1(q_1, q_1^*)$ and $\pi_1^*(v_1, v_1^*)$.

IV) Thus from I), II) and III) on the basis of the theorem of Nash it follows that there is an Curnout equilibrium on the second step of the game $\left\langle (q_1^0(t, t^*), q_1^{*0}(t, t^*)), (v_1^0(t, t^*), v_1^{*0}(t, t^*)) \right\rangle$.

Now we shall return to the first step of the game and we shall consider the game between governments. Let's show that the conditions of the theorem guarantee the existence of Nash equilibrium on the first step as well, an consequently the existence of the perfect subgame Nash equilibrium, which will make the optimum two-part trade policy.

V) In the beginning we shall show that the problem of maximization $W(t,t^*)$ by t can be considered on a segment (compact set) $X_1=[t_н, t_в]$ and the problem of maximization $W^*(t,t^*)$ by t* can be considered on a segment (compact set) $X_2=[t^*_н, t^*_в]$. This fact immediately comes from continuity and limitation of functions $q_1(t,t^*), q_1^*(t,t^*), v_1(t,t^*), v_1^*(t,t^*)$.

VI) As $p(Q), p^*(Q^*) \in C^2$; $c_1(q) \in C^2$ and $c_j^*(v) \in C^2$, then functions

$$W(t,t^*) = \int_0^{Q(t,t^*)} p(s)ds - c(q_1(t,t^*)) - c^*(v_1(t,t^*));$$

$$W^*(t,t^*) = \int_0^{Q^*(t,t^*)} p^*(s)ds - c(q_1^*(t,t^*)) - c^*(v_1^*(t,t^*))$$ are continuous.

VII) Further we shall prove concavity of functions $W(t,t^*)$ and $W^*(t,t^*)$ by t и t* accordingly.

We differentiate function $W(t,t^*)$ twice by t :

$$\frac{\partial^2 W(t,t^*)}{\partial t^2} = (p'-c'')\cdot\left(\frac{\partial q_1}{\partial t}\right)^2 + 2\cdot p'\cdot\frac{\partial q_1}{\partial t}\cdot\frac{\partial v_1}{\partial t} + (p'-c^{*}{''})\cdot\left(\frac{\partial v_1}{\partial t}\right)^2 + (p'-c'')\cdot\alpha^2\cdot\left(\frac{\partial v_1}{\partial t}\right)^2 +$$

$$+2\cdot p'\cdot\alpha\cdot\left(\frac{\partial v_1}{\partial t}\right)^2 + (p'-c^{*}{''})\cdot\left(\frac{\partial v_1}{\partial t}\right)^2 = \left(\frac{\partial v_1}{\partial t}\right)^2\cdot\left(\alpha^2\cdot(p'-c'') + 2\cdot\alpha\cdot p' + (p'-c^{*}{''})\right) =$$

$$= \left(\frac{\partial v_1}{\partial t}\right)^2\cdot\left((\alpha+1)^2\cdot p' - \alpha^2\cdot c'' - c^{*}{''}\right) \qquad (4.34)$$

We shall use the Lemma 4.2 which was previously proved by author of the project:

$$\frac{\partial q_1}{\partial t} > 0, \quad \frac{\partial v_1}{\partial t} < 0 \text{ и } \frac{\partial q_1}{\partial t} = \alpha\frac{\partial v_1}{\partial t}, \quad \alpha \in (-1,0).$$

$$\frac{\partial^2 W}{\partial t^2} = \left(\frac{\partial v_1}{\partial t}\right)^2\cdot\left((\alpha+1)^2\cdot p' - \alpha^2\cdot c'' - c^{*}{''}\right) < 0$$

Thus function $W(t,t^*)$ is concave by t. Similarly we obtain a concavity $W^*(t,t^*)$ by t*.

VIII) So, from V), VI) and VII) On the basis of the Nash theorem it follows that there is Nash equilibrium on the first step of the game $\left(t^0,t^{*0}\right)$. From IV) and VIII) it follows that in the game Γ there exists perfect subgame Nash equilibrium: $X^0 = \left\langle (e^0,\tilde{v}^0,t^0),(e^{*0},\tilde{q}^0,t^{*0}),(q_1^0(t,t^*),q_1^{*0}(t,t^*)),(v_1^0(t,t^*),v_1^{*0}(t,t^*))\right\rangle$, where

$$e^{*0} = p^*(v_1^0(t^0,t^{*0})+q_1^{*0}(t^0,t^{*0}))\cdot q_1^{*0}(t^0,t^{*0})-c\ (q_1^{*0}(t^0,t^{*0}))-t^{*0}\cdot q_1^{*0}(t^0,t^{*0}),$$

$$e^0 = p(q_1^0(t^0,t^{*0})+v_1^0(t^0,t^{*0}))\cdot v_1^0(t^0,t^{*0})-c^*(v_1^0(t^0,t^{*0}))-t^0\cdot v_1^0(t^0,t^{*0}),$$

$\tilde{v}^0 = v_1^0(t^0,t^{*0})$, $\tilde{q}^0 = q_1^{*0}(t^0,t^{*0})$. Proved.

From the given theorem we get a remarkable corollary.

Proof of Corollary 4.5. :

As the existence of the optimum policy is proved in the theorem now it is possible to return to a problem of its searching. Differentiating functions $W(t,t^*)$ by t we get:

$\dfrac{\partial\ W(t,t^*)}{\partial t} = (p'-c')\cdot\dfrac{\partial q_1}{\partial t}+(p-c^*{}')\cdot\dfrac{\partial v_1}{\partial t}$. From conditions (4.21) we have that

$$\frac{\partial\ W(t,t^*)}{\partial t} = -p'\cdot q_1\cdot\frac{\partial q_1}{\partial t}-p'\cdot v_1\cdot\frac{\partial v_1}{\partial t} = p'\cdot(q_1\cdot\frac{\partial q_1}{\partial t}+v_1\cdot\frac{\partial v_1}{\partial t}) = -p'\cdot\frac{\partial v_1}{\partial t}\cdot(\alpha\cdot q_1+v_1); \qquad (4.35)$$

From (15), Lemma 4.2 and Corollary 4.2 we get that

$$\left.\frac{\partial\ W(t,t^*)}{\partial t}\right|_{t=0} = -p'\cdot\frac{\partial v_1}{\partial t}\cdot(\alpha+1)\cdot q_1 < 0; \qquad (4.36)$$

Similarly we get that $\left.\dfrac{\partial\ W^*(t,t^*)}{\partial t}\right|_{t=0} < 0$ (4.37)

The conditions (4.36) and (4.37) imply that the optimum two-part tariff is a subsidy. Thus the optimum payment for the license is equal to flowing sales proceeds of a foreign firm:

$$e^{*0} = p^*(v_1^0(t^0,t^{*0})+q_1^{*0}(t^0,t^{*0}))\cdot q_1^{*0}(t^0,t^{*0})-c\ (q_1^{*0}(t^0,t^{*0}))-t^{*0}\cdot q_1^{*0}(t^0,t^{*0}),$$

$$e^0 = p(q_1^0(t^0,t^{*0})+v_1^0(t^0,t^{*0}))\cdot v_1^0(t^0,t^{*0})-c^*(v_1^0(t^0,t^{*0}))-t^0\cdot v_1^0(t^0,t^{*0})$$

<u>Proof of Lemma 5:</u>

Let $0 < t^* < c_2 - c_1$. There cannot be an equilibrium in which both p_1^* and p_2^* are strictly above c_2. Firm 2 does not charge less than c_2 (it would make a negative profit if it sold). Firm 1 can guarantee itself a profit as close as possible to $(c_2 - c_1) \cdot D(c_2) - t^* \cdot D(c_2)$ by charging $c_2 - \varepsilon$ (with ε small and positive). But since the market price (the minimum of the two prices) does not exceed c_2, this profit is also the highest that firm 1 can obtain. One can define the equilibrium as the limit, so $p_1^* = c_2$ and firm 1's profit is $(c_2 - c_1) \cdot D(c_2) - t^* \cdot D(c_2)$. When $p^m(c_1) < c_2$, firm 1 can charge its monopoly price without worrying about firm 2's threat. Then optimal two-part policy is the positive tariff:

$t^* = (c_2 - c_1),\ \ if\ \ c_2 \le p^m(c_1)$, in this case $\pi_2 = 0, e^* = 0, G^*_{\max} = (c_2 - c_1) \cdot D^*(c_2)$ and

$t^* = (p^m(c_1) - c_1),\ \ if\ \ c_2 > p^m(c_1)$, in this case $\pi_2 = 0, e^* = 0, G^*_{\max} = (p^m(c_1) - c_1) \cdot D^*(p^m(c_1))$.

If $t^* \notin (0; c_2 - c_1)$, then G* = 0.